Inequality and the Medium-Term Business Cycle

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Abstract

I estimate a medium-scale heterogeneous agents New Keynesian endogenous growth model for the US economy and show that inequality is an important driver of medium-term business cycles. Heterogeneity among households induces larger swings in innovation activity, due to a more volatile demand, yielding to ampler cycles. Absent endogenous growth, inequality matters only for high-frequency fluctuations but not for medium-term ones. The results are quantitatively significant and supported by empirical evidence based on cross-country data.

JEL classification: E12, E23, E24, E32, E44, O40

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1 Introduction

In this paper, I construct and estimate a Heterogeneous-Agent New Keynesian ("HANK") model with endogenous technical change to study how inequalities shape medium-term business cycles (*i.e.* fluctuations in the order of a decade). My motivation for doing so is that large recessions leave long-lasting scars on the economy, while sustained economic expansions appear to persistently shift up the production frontier[1]. However, existing HANK models have so far focused exclusively on explaining short-term fluctuations (*i.e.* fluctuations of two years duration), leaving medium-term business cycles out of their scope by construction. My quantitative analysis shows that inequalities are an important driver of business cycles at medium frequencies — and not only at short frequencies.

The model combines the main features of state of the art HANK models — idiosyncratic income risk, incomplete markets, debt limits — with the endogenous growth mechanism that links the usual high-frequency business cycle to the medium-term one, in the spirit of Comin and Gertler (2006). In addition, the model features standard mechanisms in the dynamic stochastic general equilibrium literature that are relevant to match the data, such as oligopolistic competition across firms, investment adjustment costs, wage and price rigidities, and monetary and fiscal policy.

Medium-term business cycles are shaped by the expansion of product varieties, which is the source of long-run growth in the economy and it is sustained through investment in research and development. The latter is driven by the value associated with patents ownership, which depends on the profit flow they generate (the profit channel) and the interest rates at which these flows are discounted (the discount factor channel). At the same time, households heterogeneity affects consumption and savings dynamics. Constrained agents, who consume all their income, and agents close to the borrowing limit, who engage in precautionary savings, contribute to more pronounced aggregate consumption, and smoother aggregate savings swings, relative to the complete-market benchmark. These translate in more volatile firms profits but smoother real interest rates, leaving the overall impact of inequality on the volatility of patent value a priori ambiguous. According to my estimated model, smoother discounting is not sufficient to offset more volatile profits, so

[1]Real per-capita GDP in the US settled on a lower trend after the Global Financial Crisis (2008-09) and, to date, did not recover back to pre-recessionary levels. Specularly, the boom of the late 90s saw GDP settling above trend very persistently, touching pre-90s trend levels only during the COVID-19 pandemic. Similar evidence for the Eurozone and Japan is reported in Benigno and Fornaro (2018), for Germany in Huber (2018), for for the US in Guerron-Quintana and Jinnai (2019), Garga and Singh (2020) and Cozzi et al. (2021), while evidence related to financial crises for a panel of Advanced and Emerging Economies is presented in Queralto (2020).

that patent values are more volatile under incomplete markets than under complete markets. Accordingly, market incompleteness amplify fluctuations in R&D spending, the innovation rate, and ultimately medium-term activity. To put that in context, the model suggests that the Great Recession costed \$5000 more to households due to the level of inequality in the US. Conversely, the late 90s boom benefited US households \$2000 more than it would have had were the US a equal society.

When I shut down endogenous growth, inequality has no effect on the medium-frequency behavior of the model; however, it still amplifies high-frequency business cycles. Intuitively, absent endogenous growth, there is not a mechanism linking high- and medium-frequency fluctuations, hence the amplification brought by inequality is confined to the high-frequency. This result outlines the importance of endogenous growth when assessing the link between inequality and cycles.

Besides endogenous growth, the model identifies two additional channels through which households heterogeneity has quantitatively important implications for medium-term business cycles. The first one regards precautionary savings. When I look at a model populated by hand-to-mouth households and permanent income consumers only, inequality is mostly irrelevant for fluctuations, at all frequencies. This highlights the role of agents who are close to the borrowing limit in amplifying fluctuations. Indeed, the latter engage in precautionary savings against the possibility of a future worsening of their (idiosyncratic) labor income state, reinforcing the consumption response to aggregate shocks. The second channel is instead related to the cyclicality of income risk. The literature has found the latter to be countercyclical (Storesletten et al., 2004, Guvenen et al., 2014 and Busch et al., 2022); taking that into account in the model more than doubles the baseline results. Intuitively, the cyclicality of income risk introduces time variation in precautionary savings, strengthening the consumption response to shocks[2].

I close the paper providing empirical indicative evidence in support of the main results and channels, using a large and long panel of countries, with information on national accounts data as well as income inequality. Consistently with the theoretical analysis, I find that more unequal countries display more volatile medium-term business cycles, after controlling for trade openness, manufacturing and investment shares in output, level of development and terms of trade volatility. The result is quantitatively strong and significant. A unit percentage increase in the Gini index of a country's income distribution yields to an average 1% increase in medium-term business cycle volatility. Moreover, the econometric

[2]These mechanisms are in line with the theoretical analysis of Challe and Ragot (2016).

model finds that more unequal countries also experience larger volatilities in research and development investment, consumption and physical capital investment, lending support to the main mechanism outlined in the heterogeneous agents model.

Related literature. My work is related to several strands of the economic literature. I contribute to the literature linking high-frequency fluctuations to medium-frequency ones [3]. This literature includes, among the others, Comin and Gertler (2006), Kung and Schmid (2015), Anzoategui et al. (2019), Bianchi et al. (2019), Guerron-Quintana and Jinnai (2019), Queralto (2020) and Cozzi et al. (2021). The work of Comin and Gertler (2006) is the first to propose a unified framework that allows the joint study of high and medium-frequency fluctuations. They do so augmenting a standard RBC model with expanding varieties endogenous growth à la Romer (1990) through investment in R&D and adoption. Anzoategui et al. (2019) instead build on the work of Comin and Gertler (2006) to estimate a model with R&D and adoption dynamics so as to investigate whether the productivity slowdown following the Great Recession was due to an endogenous response to the observed contraction in demand. Bianchi et al. (2019) are the first to estimate an endogenous growth DSGE model with data on R&D, with a focus on the different effect of debt and equity financing for productivity dynamics. Guerron-Quintana and Jinnai (2019) instead estimate a model with endogenous growth à la Romer (1990) and financial frictions à la Kiyotaki and Moore (2019). Queralto (2020) estimates a small open economy model with endogenous growth through expanding varieties tailored to analyze the South Korean 1997 financial crisis while Cozzi et al. (2021) assess the quantitative relevance of demand and supply factors in the US post-Great-Recession growth slowdown estimating a model with creative destruction as in Aghion and Howitt (1992) and Nuño (2011). Kung and Schmid (2015) instead introduce endogenous growth in the spirit of Romer (1990) to study the link between asset prices and aggregate risk premia and endogenous movements in long-term growth prospects. All these works are set-up in a representative-agent economy and thus abstract from the interactions between households-heterogeneity and medium-term cycles. I innovate over this literature explicitly considering such interactions and showing that inequality is an important driver of medium-term business cycles.

This paper also contributes to the literature on households heterogeneity (stemming from uninsurable idiosyncratic income uncertainty) which dates back to Laitner (1979) and Bewley (1983) from whom many extensions followed, such as Deaton (1991), Huggett (1993),

[3] For a great review of this literature see Cerra et al. (2020).

Aiyagari (1994) and Krusell and Smith (1998). More recent contributions are, among the others, Werning (2015), Guerrieri and Lorenzoni (2017) and Kaplan et al. (2018). In this literature, effort has been put in understanding how inequality changes the transmission mechanism of shocks and how it shapes macroeconomic aggregates behavior. I contribute to the latter strand. Related works are Krusell and Smith (1998), Werning (2015), Krueger et al. (2016), Guerrieri and Lorenzoni (2017), de Ferra et al. (2020) and Auclert et al. (2020). I advance the frontier showing that households heterogeneity matters for medium-term business cycles, for it alters innovation dynamics and hence an economy's growth prospects.

I contribute to the literature that studies the quantitative role of inequality for aggregate time series behavior, such as Challe et al. (2017), Berger et al. (2019), Auclert et al. (2020), Bayer et al. (2020) and Bilbiie et al. (2022). Within this literature, I am mostly related toBilbiie et al. (2022), whom estimate a dynamic stochastic general equilibrium model in the spirit of Christiano et al. (2005), Smets and Wouters (2007) and Justiniano et al. (2010) augmented with tractable households heterogeneity à la Bilbiie (2018). They find that inequality amplifies business cycles. My approach is complementary to theirs both in methodology and objective: I estimate a full blown heterogeneous agents model with endogenous growth and focus on the low-frequency component of cycles. I find that inequality is an important driver of both high-frequency and medium-term business cycles when the endogenous growth mechanism is embodied in the analysis, otherwise it matters only for the high-frequency.

My paper also contributes to the literature studying endogenous growth in heterogeneous households frameworks. Complementary to my work is Lee (2023), who rationalizes the secular increase in income inequality and productivity slowdowns in advanced economies through a perfect foresight model with endogenous growth and tractable households heterogeneity. The two works share a supply side that links innovation incentives to a profit channel. However, my work considers full cross-sectional heterogeneity while his tractable one; also, my paper focuses on the impact households heterogeneity has on medium-term fluctuations while Lee (2023) focuses on secular trends. As such the former is casted in an aggregate uncertainty environment while the latter favors a perfect foresight setting.

I also bring new elements to the classics of the endogenous growth literature, such as Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) incorporating productivity endogeneity in a dynamic stochastic general equilibrium model with uninsurable idiosyncratic risk due to market incompleteness and borrowing limits. My work is mostly related to Romer (1990) as growth takes place through expanding varieties.

Structure. The paper is organized as follows: Section 2 presents the baseline model; Section 3 discusses estimation methodology; Section 4 presents quantitative results while Section 5 proposes suggestive empirical evidence in support of the main theoretical results and mechanisms. Section 6 concludes.

2 The Model

The model consists of a closed economy inhabited by a unit mass of households, labor unions, final-good producers, intermediate-good producers, capital-good producers, innovators, a central bank and a fiscal authority. Households face idiosyncratic income risk, incomplete markets and debt limits, and as a result are heterogeneous in income, wealth and consumption. Prices and wages are sticky. Innovators carry out research and development in order to discover new varieties, which accumulate over time and give rise to endogenous growth in the economy.

2.1 Households

The economy is populated by a unit mass of households, indexed by $i \in [0,1]$, who face idiosyncratic income risk. Their labor productivity e_t^i follows a Markov process with a fixed transition matrix from $\Pi\left(e_t^i, e_{t+1}^i\right)$ from state e_t to state e_{t+1}, and stationary distribution $\pi\left(e\right)$. The average productivity level is normalized to one, *i.e.* $\sum_e \pi\left(e\right) e = 1$. Households value function is

$$V_t\left(e_t^i, a_{t-1}^i\right) = \max_{c_t, a_t} \left\{ \log\left(c_t^i\right) - \varphi \frac{L_t^{1+\varrho}}{1+\varrho} + \beta \nu_t \mathbb{E}_t V_{t+1}\left(e_{t+1}^i, a_t^i\right) \right\}, \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, c_t^i is consumption of the final good, L_t is aggregate hours worked — which are pinned down by labor demand and are the same across all households, as explained below — $\varrho > 0$ is the inverse Frisch elasticity of labor supply, $\varphi > 0$ is a scaling factor and $\log \nu_t = \rho^\nu \log \nu_{t-1} + \sigma^\nu \epsilon_t^\nu$ is a discount factor shock as in Auclert et al. (2021), with $\epsilon_t^\nu \overset{i.i.d.}{\sim} N\left(0,1\right)$. Households face an exogenous borrowing limit, $\mu g_Y^t \geq 0$, where g_Y is the balanced-growth-path growth rate of the economy[4].

Households enter period t with a_{t-1}^i bond holdings. They receive the ex-post net return r_{t-1} on these holdings, the real wage w_t net of taxes τ_t and firms profits according to

[4] The debt limit drifts with the economy's growth rate to ensure stationarity.

incidence rule $\bar{d}\left(e_t^i\right)$. The households' use of funds include consumption c_t^i and the purchase of bond holdings, a_t^i. The period budget constraint thus is

$$a_t^i + c_t^i = (1 + r_{r-1})\, a_{t-1}^i + e_t^i\,(1 - \tau_t)\, w_t L_t + d_t \bar{d}\left(e_t^i\right). \tag{2}$$

The household solves the Bellman Equation (1) subject to the period budget constraint (2) and to the debt limit $a_t^i \geq -\mu g_Y^t$. A solution to this problem is characterized by consumption $c_t\left(e_t^i, a_{t-1}^i\right)$ and asset $a_t\left(e_t^i, a_{t-1}^i\right)$ policy functions that map the households idiosyncratic productivity state e_t^i and asset holdings a_{t-1}^i to the optimal consumption and asset choice.

2.2 Labor market

The labor market structure follows standard practice in the New Keynesian literature (Erceg et al., 2000, Schmitt-Grohé and Uribe, 2005). Namely, every worker i provides $l_{t,i,k}$ hours of work to a continuum of unions indexed by $k \in [0, 1]$. Total labor effort for household i is $l_{t,i} \equiv \int l_{t,i,k} dk$. Each union k aggregates efficient units of work into a union specific task $L_{t,k} = \int l_{t,i,k} di$. A competitive labor packer then packages these tasks into aggregate labor services in a CES fashion:

$$L_t = \left(\int \left(L_{t,k}\right)^{\frac{\epsilon-1}{\epsilon}} dk\right)^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 0$ is the elasticity of substitution across labor services. The labor packer then sells these services to the intermediate goods firms at price W_t.

The union sets the common nominal wage $W_{t,k}$ per efficient unit for each of its members, and requires its members to supply labor services according to the uniform rule $l_{t,k,i} = L_{t,k}$. The union chooses $W_{t,k}$ to maximize the average utility of its members, given the aforementioned allocation rule and subject to quadratic utility costs of adjusting the nominal wage $\frac{\varsigma}{2}\int\left(\frac{W_{t,k}}{W_{t-1,k}} - 1\right)^2 dk$, that enter as an additive term in the households' utility function. In particular, the problem of the union is:

$$\max_{W_{t,k}} \mathbb{E}_t \sum_{s\geq 0} \beta^{t+s} \left[\int \left(\log\left(c_{t+s}^i\right) - \varphi \frac{L_{t+s}^{1+\varrho}}{1+\varrho}\right) di - \frac{\varsigma}{2}\left(\frac{W_{t+s,k}}{W_{t+s-1,k}} - 1\right)^2\right]$$

s.t.

$$L_{t,k} = \left(\frac{W_{t,k}}{W_t}\right)^{-\epsilon} L_t.$$

All unions set the same wage $W_{t,k} = W_t$ and all households work the same number of hours

$L_{t,i} = L_t$. This set up, as it is shown in Appendix A.2, gives rise to a wage New Keynesian Phillips Curve:

$$\pi_t^W \left(1 + \pi_t^W\right) = \eta_t^W + \kappa^W \left[\gamma L_t^{1+\varrho} - \frac{\epsilon - 1}{\epsilon} \left(1 - \tau_t\right) w_t L_t \left(C_t^*\right)^{-1}\right] + \beta \nu_t \mathbb{E}_t \left[\pi_{t+1}^W \left(1 + \pi_{t+1}^W\right)\right], \tag{3}$$

where $\pi_t^W \equiv {}^{W_t}/_{W_{t-1}} - 1$ is wage inflation, $\kappa^w \equiv {}^{\epsilon}/_{\varsigma}$ is the slope of the wage Phillips curve, $\eta_t^W = \rho^W \eta_{t-1}^W + \sigma^W \epsilon_t^W$ is a wage-markup shock with $\epsilon_t^W \overset{i.i.d.}{\sim} N(0,1)$ and $C_t^* \equiv \left(\int_i \frac{e_t^i}{\int e_t^i di} \left(c_t^i\right)^{-1} di\right)^{-1}$ is a virtual aggregate consumption, which summarizes how the distribution of consumption affects the aggregate wealth effect on labor supply. Unions set higher wages when the term in round brackets, a wedge in the labor first order condition of workers, is positive, conditional on future inflation.

2.3 Production

In this section I describe the production sector of the economy. There are four types of firms: (i) final-good producers, (ii) intermediate-good producers, (iii) capital-good producers and (iv) innovators.

2.3.1 Final-good producers

There is a continuum of measure one of monopolistically competitive final-good producers, indexed by $o \in [0,1]$. Each final good firms o produces a differentiated output Y_t^o, which is then costlessly assembled through a CES technology:

$$Y_t = \left(\int_0^1 \left(Y_t^o\right)^{\frac{1}{\mu^P}} do\right)^{\mu^P}, \tag{4}$$

where $\mu^P > 1$ and ${}^{\mu^P}/_{(\mu^P-1)}$ is the elasticity of substitution across final-goods.

Each final good firm o uses Y_{mt}^o units of intermediate-goods composite as input to produce output Y_t^o, according to the following linear technology

$$Y_t^o = Y_{mt}^o. \tag{5}$$

I assume each firm stets its nominal price P_t^o subject to Rotemberg (1982) adjustment costs $\varphi\left(P_t^o, P_{t-1}^o\right) \equiv \frac{1}{2\kappa^P(\mu^P-1)} \left(\frac{P_t^o}{P_{t-1}^o} - 1\right)^2$, where κ^P is the slope of the price New Keynesian Phillips Curve.

There exists a continuum j of measure N_{t-1} of price-setting intermediate-good oligopolists, each producing a different variety $j \in N_{t-1}$ of intermediate goods, with symmetric elasticity of substitution $\theta/(\theta-1)$ and $\theta > 1$. The variable N_{t-1} measures the stock of varieties in the economy and trends over time, determining the growth rate of the economy along the balanced growth path (BGP). Each oligopolist j produces output Y^j_{mt}. The intermediate-goods composite is the following CES aggregate of individual intermediate goods:

$$Y_{mt} = \left(\int_0^{N_{t-1}} \left(Y^j_{mt} \right)^{\frac{1}{\theta}} dj \right)^{\theta}.$$

In this setting, a final-good producer chooses price P^o_t and intermediate-goods quantity Y^j_{mt} to maximize its value:

$$J_t\left(P_{t-1}\right) = \max_{P^o_t, Y^j_{mt}} \left\{ \frac{P^o_t}{P_t} Y_{mt}\left(Y^j_{mt}, N_{t-1}\right) - \int_0^{N_{t-1}} P^j_{mt} Y^j_{mt} dj - \varphi\left(P^o_t, P^o_{t-1}\right) Y_t + \mathbb{E}_t \left[\frac{J_{t+1}\left(P^o_t\right)}{1+r_t} \right] \right\}$$

s.t.

$$\frac{Y^o_t}{Y_t} = \left(\frac{P^o_t}{P_t} \right)^{-\frac{\mu^P}{\mu^P - 1}}.$$

As it is shown in the Appendix A.3, a solution to this problem yields a price New Keynesian Phillips Curve:

$$\pi_t\left(1+\pi_t\right) = \eta^P_t + \kappa^P \left(\mu^P MC_t - 1 \right) + \mathbb{E}_t \left[\frac{1}{1+r_t} \pi_{t+1}\left(1+\pi_{t+1}\right) \frac{Y_{t+1}}{Y_t} \right], \tag{6}$$

where MC_t is the marginal cost of final-good producers and $\eta^P_t = \rho^P \eta^P_{t-1} + \sigma^P \epsilon^P_t$ is a price-markup shock with $\epsilon^P_t \overset{i.i.d.}{\sim} N(0,1)$.

2.3.2 Intermediate-good producers

In each industry j there is a single leader able to produce variety j with the highest quality level (which is normalized to one without loss of generality) and a fringe of competitors which are able to produce a version of good j of quality $1/\iota$. The parameter $\iota > 1$ captures the distance in quality between the leader and the followers. Firms combine households labor L^j_t and capital rented from capital-goods firms K^j_{t-1} in a Cobb-Douglas fashion, and share a common (stationary) stochastic aggregate productivity process $z_t = (1-\rho^z) z + \rho^z z_{t-1} + \sigma^z \epsilon^z_t$ where z is a scale parameter and $\epsilon^z_t \overset{i.i.d.}{\sim} N(0,1)$. Their production function thus is $Y^j_{mt} = z_t \left(L^j_t\right)^{1-\alpha} \left(K^j_{t-1}\right)^{\alpha}$.

Firms maximize their period profits d^j_{mt} subject to the residual demand curve they face,

$$Y^j_{mt} = \left(\frac{P^j_{mt}}{P_t}\right)^{-\frac{\theta}{\theta-1}} Y_{mt}.$$

All firms j set prices[5]

$$\frac{P^j_{mt}}{P_t} = \varpi MC^j_t, \text{ where } \varpi \equiv \min\left(\iota^{1-\alpha}, \theta\right) > 1. \tag{7}$$

The variable MC^j_t captures marginal costs of production. Therefore equation (7) implies that the leader will charge a constant markup ϖ over its marginal cost; the markup can either be θ *i.e.* the one he would have chosen in the absence of competition from the fringe, or $\iota^{1-\alpha}$ *i.e.* the highest markup the leader can charge without losing the market to its competitors.

As it is shown in the Appendix A.4, the intermediate firms demand schedules for labor and capital are, respectively,

$$(1-\alpha)\frac{P^j_{mt}}{P_t}\frac{Y^j_{mt}}{L^j_t} = \varpi w_t, \tag{8}$$

$$\alpha\frac{P^j_{mt}}{P_t}\frac{Y^j_{mt}}{K^j_{t-1}} = \varpi R^K_t, \tag{9}$$

where R^K_t is the (gross) rental rate of capital. For the problem faced by each firm is the same, they all set the same price and sell the same quantity. As it is shown in the Appendix A.1 and A.4, this yields the following expression for profits,

$$d_{mt} = \left(\frac{\varpi - 1}{\varpi}\right) MC_t N^{\theta-1}_{t-1} Y_{mt}, \tag{10}$$

i.e. profits are a fraction of intermediate-good firms output. Intuitively, profits are increasing in total demand Y_{mt} (*i.e.* market size effect) and marginal costs MC_t, and either decreasing in the elasticity of substitution $\frac{\theta}{\theta-1}$ $\left(\text{if } \theta < \iota^{1-\alpha}\right)$ or increasing in the technological distance between the leader and the followers ι $\left(\text{if } \theta > \iota^{1-\alpha}\right)$. Hence profits are procyclical.

[5] See, for example, chapter 7 of Barro and Sala-i Martin (2004) or Section 2.3 of Benigno and Fornaro (2018).

2.3.3 Capital-good firms

Capital-good producers own capital K_{t-1} and rents it out to intermediate-good producers, at the rental rate of capital R_t^K, subject to quadratic adjustment costs $\varphi^K(K_t, K_{t-1}) = \frac{1}{2\delta\epsilon^I}\left(\frac{K_t}{g_Y K_{t-1}} - 1\right)^2$. The capital law of motion is thus given by

$$K_t = I_t + (1-\delta) K_{t-1} - \varphi^K (K_t, K_{t-1}) K_{t-1},$$

where $\delta(0,1)$ is the capital depreciation rate. A capital-good producer chooses capital K_{t-1} to maximize its value:

$$J_t^K(K_{t-1}) = \max_{K_t} \left\{ R_t^K K_{t-1} - \left(K_t - (1-\delta) K_{t-1} + \varphi^K(K_t, K_{t-1}) K_{t-1}\right) + \mathbb{E}_t \frac{J_{t+1}(P_t^o)}{1+r_t} \right\}.$$

As it is shown in the Appendix A.5, a solution to this problem is characterized by standard Q-theory investment equations defining the relative price of capital and the capital Euler equation, respectively, as:

$$Q_t - 1 = \varphi_{K_t}^K (K_t, K_{t-1}) K_{t-1}, \tag{11}$$

$$\left(1 + r_{t-1} + \eta_t^I\right) Q_{t-1} = R_t^K + (1-\delta) + (Q_t - 1)\frac{K_t}{g_Y K_{t-1}} - \varphi^K(K_t, K_{t-1}) + \frac{K_t}{g_Y K_{t-1}} Q_t, \tag{12}$$

where $\eta_t^I = \rho^I \eta_{t-1}^I + \sigma^I \epsilon_t^I$ is a shock to the optimality condition for capital as in Auclert et al. (2021) and $\epsilon_t^I \overset{i.i.d.}{\sim} N(0,1)$.

2.3.4 Research and Development

Each period, there is an unbounded mass of perfectly competitive innovators $\mathfrak{h}$ that conduct R&D using the final good S_t in order to develop a new variety. Let $S_t^{\mathfrak{h}}$ be the total amount of R&D carried out by innovator $\mathfrak{h}$, ϑ_t her productivity that she takes as exogenous, $N_{t-1}^{\mathfrak{h}}$ her total stock of innovations, $\phi \in (0,1)$ the probability of variety obsolescence and $\varphi^N\left(N_t^{\mathfrak{h}}, N_{t-1}^{\mathfrak{h}}\right) = \frac{1}{\phi\epsilon^N}\left(\frac{N_t^{\mathfrak{h}}}{g_N N_{t-1}^{\mathfrak{h}}} - 1\right)$ the varieties adjustment costs. Then, the law of motion for varieties developed by innovator $\mathfrak{h}$ is

$$N_t^{\mathfrak{h}} = \vartheta_t S_t^{\mathfrak{h}} + (1-\phi) N_{t-1}^{\mathfrak{h}} - \varphi^N\left(N_t^{\mathfrak{h}}, N_{t-1}^{\mathfrak{h}}\right), \tag{13}$$

i.e. next period's stock of varieties is given by today's creation of new varieties (the first term on the right hand side of equation 13), plus the undepreciated varieties (second term on the RHS) net of adjustment costs (third term on the RHS). The innovator productivity ϑ_t is defined as

$$\vartheta_t = \chi_t N_{t-1} \left[\Psi_t^{\rho^N} S_t^{1-\rho^N} \right]^{-1}, \tag{14}$$

where $\chi_t = (1-\rho^\chi)\,\chi + \rho^\chi \chi_{t-1} + \sigma^\chi \epsilon_t^\chi$ is an exogenous disturbance to the R&D technology in the spirit of Anzoategui et al. (2019), $\chi > 0$ is a scale parameter, $\epsilon_t^\chi \overset{i.i.d.}{\sim} N(0,1)$, $\psi \in [0,1]$ is the elasticity of new varieties with respect to R&D, and $\Psi_t = K_{t-1}$ is meant to capture the level of sophistication in the economy in the spirit of Comin and Gertler (2006) and ensures stationarity. This technology features positive spillovers from the aggregate stock of varieties (innovations) $\partial\vartheta/\partial N > 0$, as in Romer (1990), and a congestion externality $\partial\vartheta/\partial S < 0$ that raises the cost of developing new varieties as the *aggregate* level of R&D raises.

Discovering a new variety j grants to the innovator exclusive rights to produce given variety. Hence, the value of such a discovery is the present discounted value of current and expected future firms' profits, *i.e.*

$$V_t = d_{mt} + (1-\phi)\,\mathbb{E}_t \left[\frac{V_{t+1}}{1+r_{t+1}} \right], \tag{15}$$

which does not depend on the specific variety j because of symmetry in equilibrium as of Section 2.3.

Since all innovators face the same problem, given the linearity of the innovator's technology and free entry, at the margin it must be that

$$\frac{1}{\vartheta_t} = (1-\phi)\,\mathbb{E}_t \left[\frac{V_{t+1}}{1+r_{t+1}} \right], \tag{16}$$

i.e. the marginal cost of R&D (the left-hand side) must be equal to its expected discounted marginal benefit (the right-hand side). The procyclicality of monopolists profits d_{mt} transfers to the cyclical pattern of firm's value V_t and, as a result, to R&D investment.

For equation (16) is independent from innovators specific characteristics, aggregation among them yields the aggregate law of motion for varieties:

$$N_t = \vartheta_t S_t + (1-\phi)\,N_{t-1} - \varphi^N (N_t, N_{t-1}). \tag{17}$$

Aggregate intermediate-good profits net of R&D expenditures are defined as

$$D_{mt} = \int_0^{N_{t-1}} d_{mt}^j dj - S_t. \tag{18}$$

2.3.5 Aggregate Production

As it is shown in the Appendix A.6, the aggregate production function is of the familiar form

$$Y_t = \left(\int_0^1 \left(Y_t^o\right)^{\frac{1}{\mu^P}} do \right)^{\mu^P} = z_t N_{t-1}^{\theta-1} K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{19}$$

where K_{t-1} is aggregate capital and L_t is aggregate labor. Equation (19) suggests that the productivity process is driven by an exogenous stationary component, z_t, and by an endogenous *trending* one, N_{t-1}, which is pinned down by R&D choices. Through this formulation, exogenous shocks have permanent effects on productivity and thus on output.

2.4 Fiscal policy

In period t, the government issues one-period nominal bonds $g_Y^t B$ and levies taxes τ_t to cover its spending needs G_t[6]. The government (balanced) budget constraint reads:

$$\tau_t w_t L_t + g_Y^{t+1} B = \left(1 + r_{t-1}\right) g_Y^t B + G_t, \tag{20}$$

where public expenditure G_t follows a first-order autoregressive process: $\frac{G_t}{g_Y^t} = \left(1 - \rho^G\right) \bar{G} + \rho^G \frac{G_{t-1}}{g_Y^{t-1}} + \sigma^G \epsilon_t^G$ where $\epsilon_t^G \overset{i.i.d.}{\sim} N(0,1)$ and $\bar{G}$ is the BGP level of public expenditure.

2.5 Monetary policy

The central bank sets the nominal policy (gross) rate i_t according to the following Taylor rule:

$$i_t = r^* + \phi_\pi \pi_t + \eta_t^{mp}, \tag{21}$$

where $\pi_t \equiv {}^{P_t}/_{P_{t-1}} - 1$ denotes price inflation, r^* is the target real interest rate, $\phi_\pi > 1$, and $\eta_t^{mp} = \rho^{mp} \eta_{t-1}^{mp} + \sigma^{mp} \epsilon_t^{mp}$ is a monetary policy shock with $\epsilon_t^{mp} \overset{i.i.d.}{\sim} N(0,1)$. Instead, the Fisher equation reads:

$$1 + r_t \equiv \frac{\left(1 + i_{t-1}\right)}{\left(1 + \pi_t\right)}. \tag{22}$$

2.6 Equilibrium

In this section I define the equilibrium concept in this economy.

[6] Government bonds B drift with the economy's growth rate g_Y to ensure stationarity.

Definition 1. *An equilibrium in this economy is a sequence of aggregate quantities Y_t, C_t, A_t, S_t, N_t,V_t, K_t, L_t, MC_t, rates π_t and π_t^W,individual level quantities c_t^i and a_t^i, prices r_t, R_t^K, w_t, Q_t, and aggregate variables T_t such that capital-goods firms optimality regarding the relative price of capital (11) and the Euler Equation for capital (12) hold, the intermediate-goods firms labor (8) and capital (9) demand schedules hold, innovators break-even according to (16), discovered varieties follow the law of motion (17), aggregate profits are defined according to (18), aggregate product is produced according to (19), the government balances its budget (20), the real interest rate obeys the Fisher rule (22) while the nominal rate is given by the Taylor rule (21), the New Keynesian Phillips curve for prices (6) and wages (3) hold, together with assets market clearing*

$$\int_i a_t\left(e_t^i, a_{t-1}^i\right) di = g_Y^{t+1} B,$$

and goods market clearing

$$Y_t = \int_i c_t\left(e_t^i, a_{t-1}^i\right) di + I_t + S_t + G_t + \varphi\left(P_t, P_{t-1}\right) Y_t,$$

given the forcing processes z_t, ν_t, χ_t,G_t, η_t^I, η_t^W, η_t^P, and η_t^{mp}, the initial wealth distribution a_{-1}^i as well as N_{-1} and K_{-1}, at any point in time t.

2.7 The growth equation

Combining the law of motion of varieties (17) with the innovators' productivity definition (14) and R&D first order condition (16) one obtains

$$g_t^N = \kappa^g \left[\left(\frac{\chi_t}{K_{t-1}^{\rho_N}}\right)^{\frac{1}{1+\rho_N}} \mathbb{E}_t\left[(1-\phi)\frac{V_{t+1}^N}{1+r_{r+1}}\right]^{\frac{\rho_N}{1-\rho_N}} + (1-\phi)\right] + \alpha^g, \tag{23}$$

where $\kappa^g \equiv \frac{\phi g^N \epsilon^N}{1+\phi g^N \epsilon^N}$ and $\alpha^g \equiv \frac{1}{1+\phi g^N \epsilon^N}$. Equation (23) links the varieties (and hence the economy's) growth rate g_t^N to the discount factor $\frac{1}{1+r_{t+1}}$, oligopolists firm value V_{t+1}^N, the exogenous disturbance to the R&D technology χ_t, and inversely to the level of sophistication of the economy K_{t-1}. Making use of the oligopolists firms value definition (15) and iterating forward Equation (23), one gets the Growth Equation in this economy, *i.e.* the equation that fully captures all the determinants of innovators' incentives to invest in research and development[7]:

[7]This is a standard equation in the endogenous growth literature; see, e.g., Barro and Sala-i Martin (2004).

$$g_t^N = \kappa^g \left[\left(\frac{\chi_t}{K_{t-1}^{\rho_N}} \right)^{\frac{1}{1+\rho_N}} \mathbb{E}_t \left[\sum_{h=1}^{\infty} (1-\phi)^{h-1} M_{t+h,t} d_{mt+h}^N \right]^{\frac{\rho_N}{1-\rho_N}} + (1-\phi) \right] + \alpha^g, \qquad \text{(GG)}$$

where $M_{t+h,t} \equiv \prod_{s=1}^{h} \frac{1}{1+r_{t+s}}$. This equation implies that two channels affect R&D incentives: a (i) discount factor channel $\{M_{t+h,t}\}_{h=1}^{\infty}$ and a (ii) profit channel $\{d_{mt+h}^N\}_{h=1}^{\infty}$. The higher the discount factors $\{M_{t+h,t}\}_{h=1}^{\infty}$, the more valuable is the future to innovators and therefore the bigger will be their investment in research and development. Also, larger expected profits $\{d_{mt+h}^N\}_{h=1}^{\infty}$ lure innovators to invest more in R&D (this is the usual market-size effect emphasized by the endogenous growth literature). Notice that in equilibrium profits d_{mt+h} are linked to total intermediate goods demand Y_{mt+h} (see Equation 10), and are thus procyclical. Intuitively, any difference in the behavior of the discount factor or in the profit channel brought by households heterogeneity, implies different speeds of varieties accumulation, and therefore different medium-term dynamics between the two model economies. Since varieties is the only trending variable, differences in their dynamics also bring permanent differences between heterogeneous agents and representative agent models.

2.8 Representative agent model

To gauge the effects of inequality for the medium-term business cycle, I will compare the baseline model with its representative-agent counterpart. The latter is obtained assuming complete markets (i.e. households have access to the complete set of Arrow-Debreu securities). As a result, individuals in this economy behave as a large representative family of permanent-income consumers where the family head ensures each member an equal marginal utility of wealth. Model equations are in the Appendix A.9.

2.9 Two-agents model

I will also compare the baseline model to a two-agents model, to elicit the effects of full cross-sectional heterogeneity, and in particular of the presence of precautionary savers. The two-agents model is populated by a mass $\lambda \in (0,1)$ of hand-to-mouth agents who consume all their labor income in each period, and a mass $1-\lambda$ of permanent income consumers who instead have access to financial markets and smooth their consumption as a result of utility maximization. Model equations are in the Appendix A.11.

3 Estimation

3.1 Estimation methodology

The model features the same set of standard shocks present in the DSGE literature (Christiano et al., 2005, Smets and Wouters, 2007 and Justiniano et al., 2010): TFP, government expenditure, monetary policy, price and wage markup, discount factor and investment shocks. In addition to these, I have introduced R&D productivity shocks in the spirit of Anzoategui et al. (2019) to capture the evolution of R&D investment over time.

I estimate the model using Beyesian methods in the Sequence Space (Auclert et al., 2021). I use quarterly data from the 1964Q1-2019Q4 on eight US time series: seven of them are standard, namely nominal GDP, investment, personal consumption expenditures, average hourly earnings, federal funds rate, inflation and hours worked, to which I add a series for research and development investment. All series are filtered with the Christiano and Fitzgerald (2003) bandpass filter to isolate fluctuations at the medium-term, defined as fluctuation within 50 years (Comin and Gertler, 2006). Additional details on the data used and observables definition are in the Appendix C. Since there are as many shocks as observables, the model is identified, is not subject to stochastic singularity, and estimation is possible.

I estimate the parameters governing the volatility and autoregressive component of shocks. I use very dispersed priors. In particular, for standard deviation coefficients I assume an inverse gamma distribution centered on 0.4 with standard deviation of 4, while for autoregressive coefficients a beta distribution centered on 0.5 and standard deviation of 0.2.

3.2 Calibrated parameters

The model has 22 parameters and are calibrated to match U.S. long-run first moments.

Regarding households parameters, the discount factor β is set to 0.947 to ensure a 1.25% real interest rate in the steady state. I calibrate the inverse of the Frisch elasticity of labor supply ϱ to 0.5, an intermediate value between micro and macro estimates (Chetty et al., 2011). The scale parameter γ in the labor disutility function is calibrated to 0.81 to normalize hours worked to be 1 in the steady state.

Table 1: Parameter values

Parameter	Description	Value				Source/Target
Households		HA	RA	TA	HA-Cyc	
β	Subjective discount factor	0.947	0.99	0.99	0.944	s.s. real interest rate of 1.25%
μ	Borrowing constraint	0				baseline scenario
ϱ	Inverse Frisch elasticity	0.5				Chetty et al. (2011)
γ	Hours scale parameter	0.81				normalize hours worked to 1 in s.s.
λ	Share of HtM households	/	/	0.3	/	Kaplan and Violante (2014)
ζ	Cyclicality of income risk	0	/	/	-0.15	Auclert et al. (2024a)
Firms						
α	Capital share	0.1				$I/Y = 6\%$
δ	Capital depreciation rate	2.5%				standard
ϕ	Obsolescence rate	2%				Caballero and Jaffe (1993)
θ	Defines el. of sub. across varieties	1.35				Broda and Weinstein (2006)
ι	Leader tech. advantage	0.11				$R\&D/Y = 6\%$
ρ^N	Elasticity of new varieties to R&D	0.37				Anzoategui et al. (2019)
χ	R&D scale parameter	0.12				2% annual growth rate
ϵ^I	Capital adj. cost	0.267				Auclert and Rognlie (2018)
κ^g	Slope of growth equation	0.001				Bianchi et al. (2019)
κ^P	Slope of price Phillips curve	0.17				Nakamura and Steinsson (2008)
κ^W	Slope of wage Phillips curve	0.003				Grigsby et al. (2021)
μ^P	Price markup	0.1				Auclert et al. (2024b)
μ^W	Wage markup	0.1				Auclert et al. (2024b)
Policy						
ϕ_π	Taylor rule inflation coefficient	1.5				standard
$\bar{G}$	s.s government expenditure	10%				U.S. post-war mean $\bar{G}/Y$
B	Government bonds	70%				U.S. post-war mean B/Y

Notes: "HA" refers to the baseline heterogeneous agents model, "RA" to its representative agent counterpart, "TA" to the two-agents model and "HA-Cyc" to the heterogeneous agents model with cyclical income risk. Empty values are to be intended calibrated as in the HA model.

The income process instead is defined as $\log e_{it} = \rho^e \log e_{it-1} + \sigma^e \epsilon_{it}$ and it is discretized using the Rouwenhorst method over 11 states (Kopecky and Suen, 2010). The persistence parameter ρ^e is set to 0.966 and the standard deviation σ^e to 0.92, which help to match the quarterly persistence and standard deviation of the persistent component of log income in the US. The number of points on the asset grid is set to 500. I first take a uniformly spaced grid. Then I double-exponentially transform such a grid in order to obtain an asset grid where there are more points near the borrowing limit (because this is where the policy function is more nonlinear in assets). I further set the borrowing limit $\mu = 0$ as in McKay et al. (2016).

Turning to production and R&D parameters, I set the capital share α to 0.1 in order to yield a investment (in physical capital) to GDP ratio of 11%. The depreciation rate of capital δ is set to 2.5%, a standard value. The investment adjustment costs parameter ϵ^I is set to 0.267 so that the elasticity of investment to $Q$is in line with estimates in Gilchrist and Himmelberg (1995). The obsolescence rate ϕ is set to 2%, in line with evidence in Caballero and Jaffe (1993). In the literature there is not a consensus about the size of the R&D to GDP ratio and the range goes from 1% (Comin and Gertler, 2006) to 16% (Bilbiie et al., 2012), as noted in Guerron-Quintana and Jinnai (2019). I take a value of 6% as the benchmark. In order to hit that target, I calibrate the leader technological advantage parameter ι to 0.11 which in turn yields a 11% markup, which is in line with common estimates (Basu and Fernald, 1997). According to the equilibrium pricing rule (7) the prevailing markup is the constrained one. Regarding the elasticities of new varieties with respect to R&D, ρ^N, I set it to 0.37, in line with the estimated value in Anzoategui et al. (2019). The scale parameter χ instead is set to 0.12 which yields a 2% annual growth rate for the economy. The varieties adjustment cost parameter ϵ^N is calibrated to induce a slope for the growth equation κ^g of 0.0015, *i.e.* a fourth of the elasticity of capital to Q, in order to capture the more sluggish R&D dynamics, consistent with Bianchi et al. (2019). To calibrate the slope of the Phillips curves, I follow Auclert et al. (2024b) and use the formula implied by the Calvo model: $\kappa = \frac{1}{1+\Gamma}\left(1 - \frac{1}{1+r}\left(1 - freq\right)\right) freq\left(1 - freq\right)^{-1}$ where $freq$ is the frequency of price change and Γ is a real rigidity parameter. For prices, I convert the frequency of sale price changes in Nakamura and Steinsson (2008) to quarterly, giving $freq = 0.22$. For wages I take Grigsby et al. (2021)'s $freq = 0.33$. I apply to both a real rigidity coefficient of $\Gamma = 5$, obtaining $\kappa^P = 0.17$ and $\kappa^W = 0.003$. Also, wage and price markups μ^W, μ^P are set to 10% as in Auclert et al. (2024b).

Parameters regarding monetary and fiscal policy are as follows: the monetary policy inflation reaction coefficient ϕ_π is set to a standard value of 1.5 while the steady state value of public expenditure $\bar{G}$ is set to 0.1 while that of government bonds B to 0.7 consistent with U.S. post-war mean estimates of expenditure and debt to GDP ratios. Parameter values are summarized in Table 1.

Representative agent model. The representative agent model is calibrated as the baseline model, with the exception of the discount factor β which is pinned down by the households Euler equation for bonds and is equal to 0.99 to induce the same steady state real interest rate of $r = 1.25\%$.

Two-agents model. The two-agents agent model is calibrated as the baseline model, with the exception of the discount factor β which is pinned down by the unconstrained households Euler equation for bonds and is equal to 0.99 to induce the same steady state real interest rate of $r = 1.25\%$. The share of hand to mouth households λ is set to 0.3, in line with estimates in Kaplan and Violante (2014).

Solution method. The baseline model is non-stationary due to cointegration in the varieties variable N_t; thus the model is firstly detrended and then solved around the balanced-growth-path. Details are in the Appendix A.8. The steady state of the model is solved with standard methods for incomplete markets economies with idiosyncratic risk. It involves a backward iteration on the marginal value function to obtain the steady state policy function. This is done with the endogenous gridpoints method of Carroll (2006). Instead, to solve the model under aggregate uncertainty and perform Bayesian estimation, I rely on the Sequence Space Jacobian method set out in Auclert et al. (2021). Shocks parameters estimates are discussed in the Appendix D.

4 Inequality and the medium-term business cycle

In this section I analyze the impact inequality has on medium-term business cycles. In order to do so, first I extract the sequence of historical shocks that allow the baseline model to match the observed time series over the period 1966Q1-2017Q4[8]. Second, I feed the representative-agent-equivalent version of the baseline model with the same historical shocks and compare the evolution of output. Comparing this two models allows to detect the role of inequality in driving medium-term business cycles. Finally, I feed with the same shock sequence other counterfactual models to dissect various mechanisms behind the main result.

[8]Two years of data at the beginning and end of the original sample are dropped because are poorly estimated by the bandpass filter. See Christiano and Fitzgerald (2003).

4.1 Main result

Figure 1 shows the model's generated medium-term business cycle in output, and its medium-frequency and high-frequency components[9], for both the baseline model and its representative-agent counterpart[10].

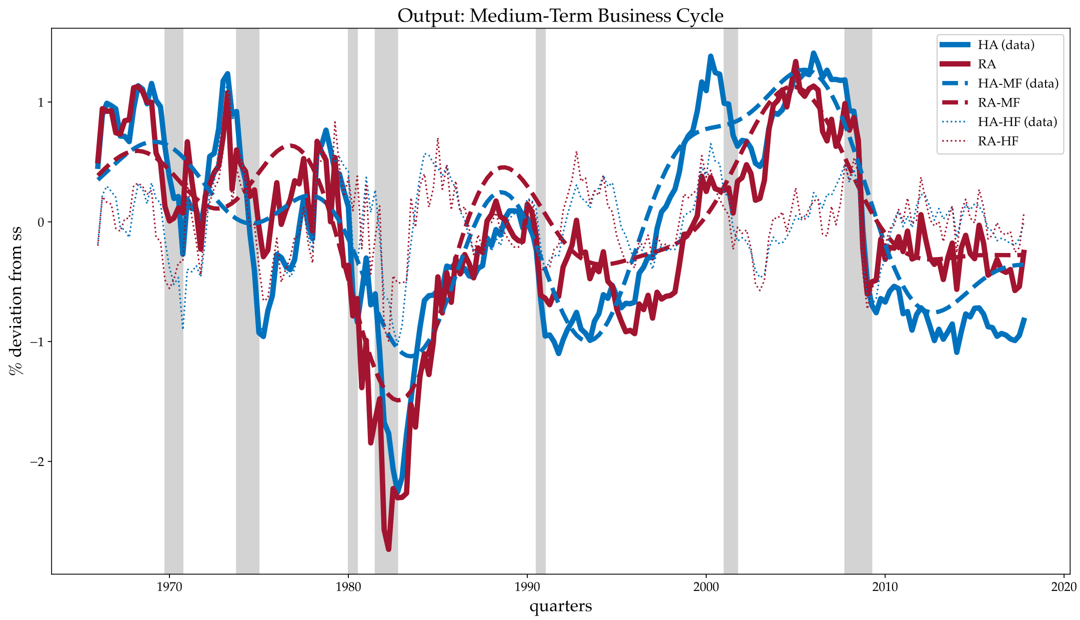

Figure 1: Simulated model's (HA in blue and RA in red) time series at the medium-term business cycle frequency (MT, fluctuations within 2-200 quarters, solid lines), medium-frequency (MF, fluctuations within 32-200 quarters, dashed lines) and high-frequency (HF, fluctuations within 2-32 quarters, dotted lines), extracted with the Christiano and Fitzgerald (2003)'s bandpass filter. Grey bars denote NBER recessions. Time period is 1966Q1 to 2017Q4.

The baseline model displays more volatile cycles in all frequencies. In particular, the medium-term business cycle is overall 11% more volatile when households heterogeneity is taken into account. This means that, looking at the oil crises of the 70s, output would have fallen below trend by about 0.2% in the absence of inequality, against the actual fall of almost 1%. The recession of the 90s related to the oil shock caused by the Iraqi invasion of Kuwait would have been half as deep as it had been, would the US society be more equal. Also, looking at the medium-frequency component of the business cycle, the recession caused a deeper and more persistent drop in medium-term growth compared to the equal-society counterfactual. This behavior of output is similar in the post Global

[9]The medium-term business cycle is defined as fluctuations within 2 and 200 quarters, the medium-frequency as fluctuations within 32 and 200 quarters while the high-frequency as fluctuations within 2 and 32 quarters. The latter is the standard definition of business cycle. See Comin and Gertler (2006).

[10]Time series for the components of output are reported in Figure 5 in the Appendix B.

Financial Crisis period, where growth was impaired more severely and for longer compared to the equal-society counterfactual. In numbers, this means that the cumulative per capita cost of the Global Financial Crisis would have been $5000 smaller than it had, were the United States a equal society. Of course, more volatile cycles also mean that booms can be more peaked. In particular, the dot-com boom of the early 2000s is not much of a boom in the equal-society counterfactual. This evidence suggests that inequality is an important driver of medium-term business cycles. Let me now discuss why.

Table 2: Inequality and the medium-term business cycle across different model specifications

		Output standard deviation (HA/RA)		
		MT	MF	HF
1.	Baseline	11	12	17
2.	Exogenous Growth	5	3	16
3.	Two-agents model	2	2	1
4.	Counter-cyc. inc. risk	26	30	28

Notes: Entries refer to percentage deviations of output standard deviations and output growth rate standard deviations between the HA and RA model economies under different specifications. All model variants are fed with the historical shocks extracted for the baseline model. HF stands for "high frequency" (HF: 2-32), MF for "medium frequency" (MF: 32-200) and MT for "medium-term business cycle" (MT: 2-200). Simulation period is 1966Q1-2017Q4.

Mechanism. The low frequencies of the model are shaped mostly by varieties accumulation, as this is the most persistent accumulating variable in the model[11]. The Growth Equation (GG) shows that varieties accumulation is driven by a profits channel and a discount factor channel, which are both affected by inequality. Households heterogeneity affects consumption/savings behavior, with a departure from the representative agent economy. In particular, as it is shown in Kaplan et al. (2018), aggregate consumption in the heterogeneous agents economy is more responsive to income changes rather than interest rate changes. Also, the presence of households who are borrowing constrained and behave "hand-to-mouth" (i.e. consume all their income in each period) and households close to the borrowing limit who form precautionary savings, make aggregate consumption more volatile. This can be seen looking at the consumption time series in Figure 5 in the Appendix B. Consumption volatility is 6% higher in the heterogeneous agents model. In terms of innovation dynamics and the medium-term, this has two effects: first, due to nominal

[11] See Figure 6 in the Appendix B.

rigidities, output is partly demand-driven and thus the higher volatility in consumption transfers to output and firms profits; second, the lower sensitivity of consumption to the real rate and the higher volatility of the former imply that the latter experiences smoother fluctuations. Quantitatively, the profits channel dominates and households heterogeneity amplifies innovators investment in R&D and, hence, the medium-term business cycle.

4.2 Counterfactuals

In this section I discuss how the baseline result changes when, alternatively, the endogenous growth mechanism is shut down, cross-sectional heterogeneity is reduced to just a mass of constrained agents and a complementary mass of unconstrained ones and when income risk is allowed to change over the cycle.

Exogenous growth. In order to do investigate the role of R&D for the main result, I feed the estimated historical shocks to the exogenous growth counterfactual of the baseline model. The business cycle for both HA and RA variants is shown in Figure 2 and summary statistics in Table 2. Absent endogenous growth, the medium-term business cycle is almost

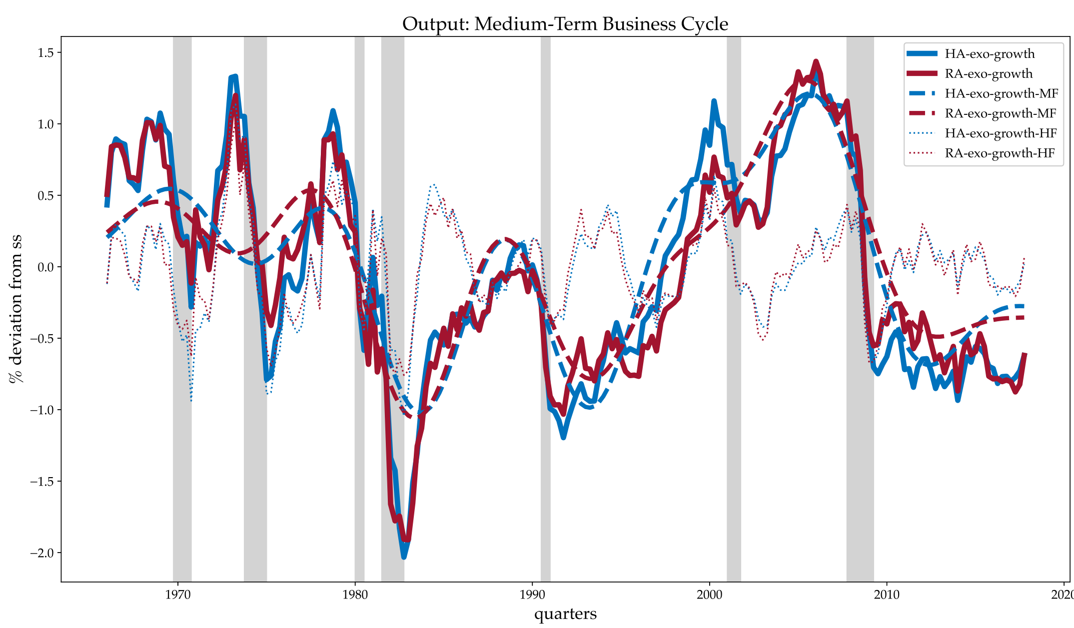

Figure 2: Simulated exogenous growth model's (heterogeneous agents "HA-exo" in blue and representative agent "RA-exo" in red) time series at the medium-term business cycle frequency (MT, fluctuations within 2-200 quarters, solid lines), medium-frequency (MF, fluctuations within 32-200 quarters, dashed lines) and high-frequency (HF, fluctuations within 2-32 quarters, dotted lines), extracted with the Christiano and Fitzgerald (2003)'s bandpass filter. Grey bars denote NBER recessions. Time period is 1966Q1 to 2017Q4.

undistinguishable between the heterogeneous agents and representative agent models. The difference in volatilities drops to 4.6%. Looking at the medium- and high-frequencies, it is possible to notice that much of the reduction in the business cycle comes from a compression of medium-frequency swings rather than high-frequency ones. Indeed, the difference in volatilities at the medium-frequency has drooped from 12% to 3% while that at the high-frequency is mostly unchanged (17% in the baseline against 16% in the exogenous growth model). This is in line with the fact that the endogenous growth block is responsible mainly for the low-frequency behavior of the model. This analysis implies that research and development is an important channel for assessing the role of inequality for low-frequency fluctuations: it makes inequality quantitatively relevant for the medium-term business cycle and suggests the two phenomena should be studied jointly for they are closely intertwined.

Two-agents model. Comparing the two-agents model to the representative agent one allows to elicit the quantitative role of full cross-sectional heterogeneity for the main result. Figure 3 shows the business cycles for the two-agents and representative agent models. The cycles in the two models are undistinguishable, at every frequency. In particular,

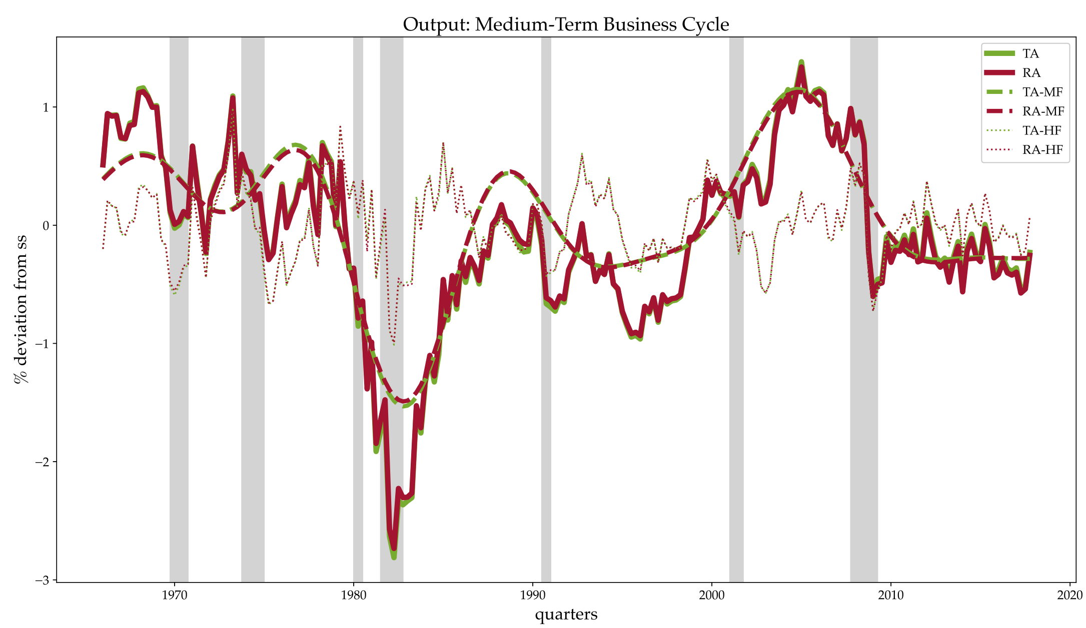

Figure 3: Simulated model's (two-agents model "TA" in green and representative agent "RA" in red) time series at the medium-term business cycle frequency (MT, fluctuations within 2-200 quarters, solid lines), medium-frequency (MF, fluctuations within 32-200 quarters, dashed lines) and high-frequency (HF, fluctuations within 2-32 quarters, dotted lines), extracted with the Christiano and Fitzgerald (2003)'s bandpass filter. Grey bars denote NBER recessions. Time period is 1966Q1 to 2017Q4.

the difference in volatilities drops to 2%. This suggests that the role of agents close to the borrowing limit, whom are not considered in the two-agents model, is quantitatively relevant for assessing the role of inequality in business cycles. This is in line with findings in Auclert et al. (2024b).

Countercyclical income risk. The baseline model features a constant income-risk; however the empirical literature has found evidence that income-risk is countercyclical (Storesletten et al., 2004, Guvenen et al., 2014 and Busch et al., 2022). If during recessions there is a higher probability of receiving bad income draws, agents will form countercyclical precautionary savings, implying an even more volatile consumption path. This might have consequences for the quantitative role of inequality for business cycles. To investigate this claim, I assume an allocation rule as in Werning (2015) and Auclert and Rognlie (2018). In particular, labor income for household i net of taxes now reads

$$\frac{\left(e_t^i\right)^{\zeta \log((1-\tau_t)w_t L_t)}}{\mathbb{E}_t\left[\left(e_t^i\right)^{1+\zeta \log((1-\tau_t)w_t L_t)}\right]}\left(1-\tau_t\right) w_t L_t,$$

where ζ controls the cyclicality of income risk. The above formulation implies that changes

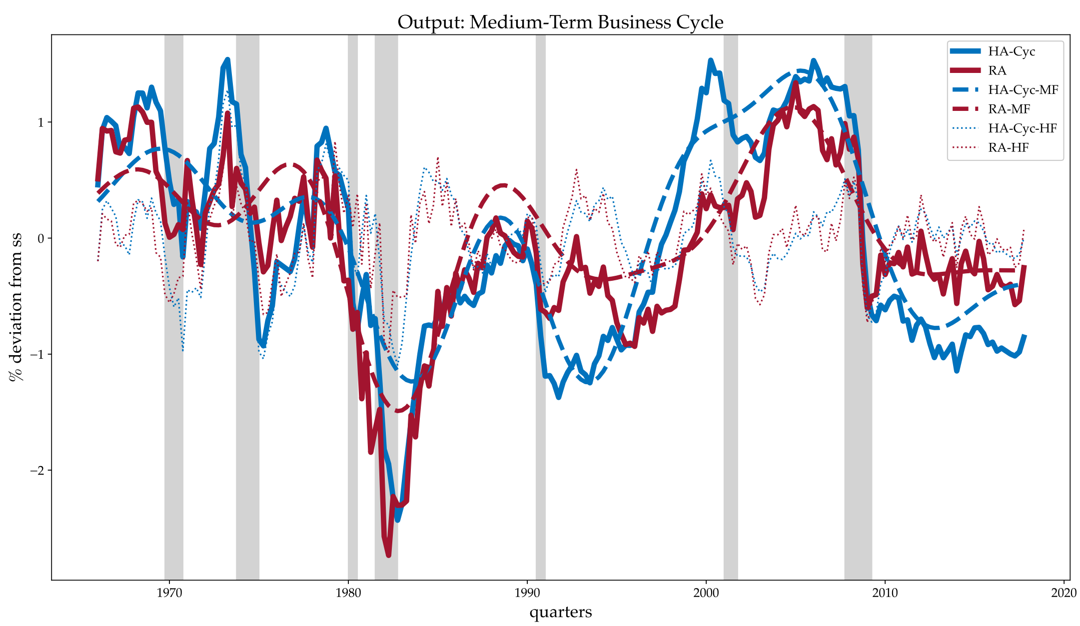

Figure 4: Simulated model's (heterogeneous agents countercyclical income risk "HA-Cyc" in blue and representative agent "RA" in red) time series at the medium-term business cycle frequency (MT, fluctuations within 2-200 quarters, solid lines), medium-frequency (MF, fluctuations within 32-200 quarters, dashed lines) and high-frequency (HF, fluctuations within 2-32 quarters, dotted lines), extracted with the Christiano and Fitzgerald (2003)'s bandpass filter. Grey bars denote NBER recessions. Time period is 1966Q1 to 2017Q4.

in aggregate labor demand L_t have different impacts along the income distribution, as governed by the parameter ζ. When ζ is negative, income risk increases during downturns and viceversa when it is positive. The case of $\zeta = 0$ brings us back to the baseline scenario of acyclical income risk. I focus on the case of countercyclical income risk, and calibrate $\zeta = -0.15$ as in Auclert et al. (2024a). Figure 4 shows business cycles for the heterogeneous agents economy with countercyclical income risk and for the representative agent model. The difference in volatility is now 26%, against 11% in the baseline case. The larger volatility in consumption amplifies the effects of the profits and discount factor channels in the Growth Equation (GG), explaining the higher volatility in the medium-term business cycle.

5 Empirical evidence

In this section I present some indicative evidence that the positive relationship between inequality and business cycles volatility, as well as the main mechanism identified by the model, are borne out by the data. The objective of this analysis is not to detect a robust and systematical causal relationship between inequality and cycles volatility, rather it is to provide suggestive evidence that speaks to the main findings of the previous section.

I combine cross-country national accounts data from the World Development Indicators of the World Bank, with inequality measures from the World Inequality Database. The former is currently the dataset with the longest time series for inequality that is comparable across countries[12]. I focus on the Gini index of the income distribution, which is a commonly studied inequality measure. To map as closely as possible from my heterogeneous agents model to the data, inequality is computed over pre-tax income, which is the relevant source of inequality in the former. The model I estimate is defined as a weighted cross-country regression of the type

$$\mathbf{Y} = \alpha + \beta \mathbf{Gini} + \gamma' \mathbf{X} + \epsilon, \tag{24}$$

where $\mathbf{Y}$ is a vector containing volatilities of output medium-term business cycles for each country, $\mathbf{Gini}$ is a vector containing the Gini index of the income distribution for each country and $\mathbf{X}$ is a vector of control variables. Each country's observation is weighted by its average GDP in constant 2015 US dollars. I am interested in the coefficient β which captures the elasticity of medium-term business cycle volatility to inequality. I rely on the growth volatility literature to select relevant controls (see Levine and Renelt, 1992, Ramey and Ramey, 1995 and Malik and Temple, 2009). In particular, the vector $\mathbf{X}$ contains

[12] Additional info on the data are in the Apendix (C).

country's trade openness, manufacturing share in GDP, investment share in GDP, initial population level, initial GDP at purchasing power parity levels and the terms of trade standard deviation.

Table 3: Inequality and the medium-term business cycle: cross-country evidence

	Output	R&D	Consumption	Investment
Gini	1.05**	.92*	.94*	.83**
	(.45)	(.59)	(.53)	(.39)
Trade Open.	-.37	.13	.12	.14
	(.23)	(.19)	(.18)	(.12)
Manufacturing	.01	.01	-.04	-.01
	(.02)	(.56)	(.03)	(.02)
Investment	.93***	-.73	.49	.27
	(.29)	(.61)	(.35)	(.31)
Population	-2.92**	-3.4***	-1.72*	-1.66*
	(1.45)	(1.11)	(1.09)	(.88)
GDP PPP	-1.8***	-2.59***	-1.97***	-2.22***
	(.45)	(.93)	(.54)	(.46)
ToT s.d.	.12	.34*	.19*	.08
	(.09)	(.18)	(.10)	(.09)
R^2	.69	.63	.64	.66
N	117	82	116	113

Notes: Standard errors are in parentheses. * indicates significance at 10%, ** at 5% and *** at 1% level, respectively.

Results are reported in Table 3. The first column shows that a unit percentage increase in the Gini index is associated with an average increase of 1.05% in a country's output medium-term volatility. To put this in context, going from a relatively equal country such as Sweden, which features a Gini index of .36, to Italy, a relatively less equal country with a Gini index of .46, should be associated with an average 30% increase in output volatility. The positive relationship is significant at the 3% level and consistent with the main result in the previous section. Also, higher initial levels of population and GDP, which are proxies for development, are significantly associated with smoother cycles. In addition, a larger share of investment in GDP — which is a volatile component of output — is intuitively associated with ampler cycles. These findings are in line with previous literature studying the determinants of growth volatility.

I then turn to the analysis of the empirical plausibility of the channel through which inequality yields to more volatile medium-term business cycles, i.e. whether research and development, consumption and investment are more volatile in more unequal societies. The second, third and fourth columns of Table 3 report results of estimating model (24) with the dependent variable defined as either R&D , consumption or investment. The relationship between inequality and R&D, consumption and investment is positive and significant, supporting the idea that more unequal countries display more volatile consumption, innovation and investment patterns, yielding to ampler fluctuations in output.

To summarise, using cross-country data in this section I have estimated a positive and significant relationship between inequality and business cycles, in support of the prediction of my quantitative heterogeneous agents model. The data also support the mechanism suggested by the model, namely that more unequal countries display larger volatilities in consumption, R&D and investment. This evidence is not conclusive on the role of inequality for fluctuations and its channels, and requires further study, however it lends support to theoretical predictions.

6 Conclusion

In this paper I studied the link between households heterogeneity and the medium-term business cycle. I did so estimating on US data a heterogenous-agents New Keynesian model in which growth arises endogenously in the expanding variety fashion through investment in R&D.

I found inequality to be an important driver of medium-term business cycles. Heterogeneity among households induces larger swings in innovation activity, due to a more volatile demand, yielding to ampler cycles. Absent endogenous growth, inequality matters only for high-frequency fluctuations but not for medium-term ones. When I look at a model populated by hand-to-mouth households and permanent income consumers only, inequality is mostly irrelevant for fluctuations, at all frequencies. This highlights the role of agents who are close to the borrowing limit in amplifying fluctuations. Also, encompassing countercyclicality in income risk makes inequality even more relevant for cycles.

I provided empirical indicative evidence in support of the main result and channels, using a large and long panel of countries, with information on national accounts data as well as income inequality. Consistently with the theoretical analysis, I find that more unequal countries display more volatile medium-term business cycles. The result is quantitatively strong and significant. In line with the mechanism identified by the model, the data suggest that more unequal countries also experience more volatile expenditure in research and development, consumption and investment.

This work has focused on inequality among households and its role for innovation dynamics and medium-term business cycles. Another promising avenue is to study how inequality among innovators and entrepreneurs affect innovation. Also, in this paper I have focused on the impact of micro-heterogeneity on macroeconomic aggregates; an interesting question to study is whether macroeconomic aggregates themselves affect inequality and why. These are promising areas I plan to pursue in the future.

The way I introduced endogenous growth in the model is intentionally stylized; it would be useful to explore different innovation settings, following the firm dynamics literature, and understand how those influence the results in the paper. The framework built in this paper is well suited for the analysis of optimal monetary policy and the study of fiscal stabilizers. Given the role inequality has in amplifying depth and persistence of downturns due to more volatile demand, policies targeted at keeping demand high during times of distress might be welfare improving. These are promising areas I leave to future research.

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Appendix

A Appendix to the baseline model

A.1 Final good firms

The problem of the final good firms is

$$\min_{Y^j_{mt}} \int_0^{N_{t-1}} P^j_{mt} Y^j_{mt} dj \tag{25}$$

s.t.

$$Y^o_t = \left(\int_0^{N_{t-1}} \left(Y^j_{mt} \right)^{\frac{1}{\theta}} dj \right)^{\theta}$$

Let MC_t be the Lagrange multiplier associated to the production constraint, then the first order condition with respect to intermediate good quantity Y^j_{mt} is

$$P^j_{mt} = MC_t \theta \left(Y^o_t \right)^{\frac{\theta-1}{\theta}} \frac{1}{\theta} \left(Y^j_{mt} \right)^{\frac{1}{\theta}-1} \tag{26}$$

while that with respect to the Lagrange multiplier simply implies

$$Y^o_t = \left(\int_0^{N_{t-1}} \left(Y^j_{mt} \right)^{\frac{1}{\theta}} dj \right)^{\theta}.$$

Instead, symmetric pricing $P^j_{mt} = P_{mt}$ and production $Y^j_{mt} = Y_{mt}$ imply that the intermediate goods quantity amounts to

$$Y_{mt} = \frac{Y^o_t}{N^{\theta}_{t-1}}. \tag{27}$$

Using the latter into (26) yields an expression for the marginal cost:

$$MC_t = \frac{P_{mt}}{N^{\theta-1}_t}. \tag{28}$$

A.2 Wage New Keynesian Phillips Curve

The problem of the union is

$$\max_{W_{t,k}} \mathbb{E}_t \sum_{s\geq 0} \beta^{t+s} \left[\int \left(\log\left(c^i_{t+s}\right) - \varphi \frac{L_{t+s}^{1+\varrho}}{1+\varrho} \right) di - \frac{\varsigma}{2} \left(\frac{W_{t+s,k}}{W_{t+s-1,k}} - 1 \right)^2 \right]$$

s.t.

$$L_{t,k} = \left(\frac{W_{t,k}}{W_t} \right)^{-\epsilon} L_t, \tag{29}$$

$$W_t = \left(\int W_{kt}^{1-\epsilon} dk \right)^{\frac{1}{1-\epsilon}}.$$

The total earnings for household i working for union k are

$$\begin{aligned} z_{it} &= (1-\tau_t) \frac{W_{kt}}{P_t} e^i_t L_{kt} \\ &\overset{\text{by (29)}}{=} (1-\tau_t) \frac{1}{P_t} \int \left(W_{kt} e^i_t \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} L_t \right) dk, \end{aligned}$$

and the envelope condition gives $\frac{\partial c^i_t}{\partial W_{kt}} = \frac{\partial z^i_t}{\partial W_{kt}}$. Using $W_{kt} = W_t$ I get

$$\begin{aligned} \frac{\partial z^i_t}{\partial W_{kt}} &= (1-\tau_t) \left[\frac{1}{P_t} e^i_t \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} L_t - \epsilon \frac{1}{P_t} e^i_t \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} L_t \right] \\ &= (1-\tau_t) \frac{1}{P_t} e^i_t \left[L_{kt} - \epsilon \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} L_t \right] \\ &= (1-\tau_t) \frac{1}{P_t} e^i_t L_{kt} (1-\epsilon). \end{aligned}$$

Households total hours worked are $L_{it} = \int L_{ikt} dk = \int \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} L_t dk$, thus

$$\frac{\partial l^i_t}{\partial W_{kt}} = -\epsilon \frac{L_{kt}}{W_{kt}}.$$

Let $u(c)$ and $v(L)$ be, respectively the households utility from consumption and disutility from labor functions; then the union's first order condition for W_{kt} is

$$\int \left(\frac{\partial u}{\partial c_t^i} \frac{\partial c_t^i}{\partial W_{kt}} - \frac{\partial v}{\partial l_t^i} \frac{\partial l_t^i}{\partial W_{kt}} \right) di - \varsigma \left(\frac{W_{kt}}{W_{kt-1}} - 1 \right) \frac{1}{W_{Kt-1}} - \beta \nu_t \varsigma \left(\frac{W_{kt+1}}{W_{kt}} - 1 \right) \left(-W_{kt+1} W_{kt}^{-2} \right) = 0.$$

In equilibrium $W_{kt} = W_t$ and $L_{kt} = L_t$. Defining wage inflation as $\pi^W \equiv \frac{W_t}{W_{t-1}} - 1$, after multiplying the expression above by W_t, the union's first order condition can be rewritten as

$$\pi_t^W \left(1 + \pi_t^W \right) = \kappa^W \int \left[v' \left(l_t^i \right) - \frac{\epsilon - 1}{\epsilon} \frac{\partial z_t^i}{\partial l_t^i} u' \left(c_t^i \right) \right] di + \beta \nu_t \mathbb{E}_t \left[\pi_{t+1}^W \left(1 + \pi_{t+1}^W \right) \right]. \quad (30)$$

In equilibrium $l_t^i = L_{kt} = L_t$ and

$$\frac{\partial z_t^i}{\partial l_t^i} = e_t^i \left(1 - \tau_t \right) \frac{W_t}{P_t},$$

which allows to rewrite (30) as in the main text (Equation 3):

$$\pi_t^W \left(1 + \pi_t^W \right) = \eta_t^W + \kappa^W \left[\gamma L_t^{1+\varrho} - \frac{\epsilon - 1}{\epsilon} \left(1 - \tau_t \right) w_t L_t \left(C_t^* \right)^{-1} \right] + \beta \nu_t \mathbb{E}_t \left[\pi_{t+1}^W \left(1 + \pi_{t+1}^W \right) \right]$$

where $w_t \equiv \frac{W_t}{P_t}$ is the real wage, $\kappa^w \equiv \epsilon/\varsigma$ is the slope of the wage Phillips curve, $\eta_t^W = \rho^W \eta_{t-1}^W + \sigma^W \epsilon_t^W$ is a wage-markup shock with $\epsilon_t^W \overset{i.i.d.}{\sim} N(0,1)$ and $C_t^* \equiv \left(\int_i \frac{e_t^i}{\int e_t^i di} \left(c_t^i \right)^{-1} di \right)^{-1}$ is a virtual aggregate consumption.

A.3 Price New Keynesian Phillips Curve

Final good firms face Rotemberg (1982) adjustment costs $\varphi \left(P_t^o, P_{t-1}^o \right) \equiv \frac{1}{2\kappa^P (\mu^P - 1)} \left(\frac{P_t^o}{P_{t-1}^o} - 1 \right)^2$, with derivatives:

$$\varphi_{P_t^o} \left(P_t^o, P_{t-1}^o \right) \equiv \frac{1}{\kappa^P \left(\mu^P - 1 \right)} \left(\frac{P_t^o}{P_{t-1}^o} - 1 \right) \frac{1}{P_{t-1}^o},$$

$$\varphi_{P_t^o} \left(P_{t+1}^o, P_t^o \right) \equiv - \frac{1}{\kappa^P \left(\mu^P - 1 \right)} \left(\frac{P_{t+1}^o}{P_t^o} - 1 \right) \frac{P_{t+1}^o}{\left(P_t^o \right)^2}.$$

The firm problem is

$$J_t\left(P_{t-1}\right) = \max_{P_t^o, Y_{mt}^j} \left\{ \frac{P_t^o}{P_t} Y_{mt}\left(Y_{mt}^j, N_{t-1}\right) - \int_0^{N_{t-1}} P_{mt}^j Y_{mt}^j dj - \varphi\left(P_t^o, P_{t-1}^o\right) Y_t + \mathbb{E}_t\left[\frac{J_{t+1}\left(P_t^o\right)}{1+r_t}\right] \right\}$$

s.t.

$$\frac{Y_t^o}{Y_t} = \left(\frac{P_t^o}{P_t}\right)^{-\frac{\mu^P}{\mu^P-1}},$$

$$Y_{mt} = \left(\int_0^{N_{t-1}} \left(Y_{mt}^j\right)^{\frac{1}{\theta}} dj\right)^{\theta} \equiv \psi\left(Y_{mt}^j\right).$$

Let η_t be the Lagrange multiplier on the production constraint. The first order condition for intermediate goods quantity Y_{mt}^j is

$$\frac{P_t^o}{P_t}\psi'\left(Y_{mt}^j\right) - P_{mt}^o + \eta_t\left(\frac{1-\mu^P}{\mu^P}\right)\left(\frac{\psi\left(Y_{mt}^j\right)}{Y_t}\right)^{\frac{1-\mu^P}{\mu^P}-1}\psi'\left(Y_{mt}^j\right) = 0.$$

This can be rewritten as

$$\underbrace{\left[\frac{P_t^o}{P_t} + \eta_t\left(\frac{1-\mu^P}{\mu^P}\right)\left(\frac{\psi\left(Y_{mt}^j\right)}{Y_t}\right)^{\frac{1-\mu^P}{\mu^P}}\right]}_{MC_t}\psi'\left(Y_{mt}^j\right) = P_{mt}^o$$

where MC_t denotes the marginal cost. Notice that in equilibrium all firms set the same price, $P_t^o = P_t$ and by (4) and (5), $Y_t = \psi\left(Y_{mt}^j\right)$, thus

$$MC_t = 1 + \eta_t\left(\frac{1-\mu^P}{\mu^P}\right). \tag{31}$$

Instead, the first order condition with respect to the price P_t^o is

$$\frac{1}{P_t}\psi\left(Y_{mt}^j\right) - \varphi_{P_t^o}\left(P_t^o, P_{t-1}^o\right) Y_t + \frac{1}{1+r_t}\frac{\partial J_{t+1}\left(P_t^o\right)}{\partial P_t^o} - \frac{\eta_t}{P_t} Y_t = 0,$$

and the envelope condition is

$$\frac{\partial J_t\left(P_{t-1}^o\right)}{\partial P_{t-1}^o} = -\varphi_{P_{t-1}^o}\left(P_t^o, P_{t-1}^o\right) Y_t.$$

Combining the two I obtain

$$\frac{1}{P_t}\psi\left(Y_{mt}^j\right) - \varphi_{P_t^o}\left(P_t^o, P_{t-1}^o\right) Y_t - \frac{1}{1+r_t}\varphi_{P_t^o}\left(P_{t+1}^o, P_t^o\right) Y_{t+1} - \frac{\eta_t}{P_t} Y_t = 0.$$

Multiplying the last expression by P_t^o, using the partial derivatives of φ and symmetric pricing $P_t^o = P_t$, together with Equation (31), I get

$$\pi_t\left(1+\pi_t\right) = \eta_t^P + \kappa^P\left(\mu^P MC_t - 1\right) + \mathbb{E}_t\left[\frac{1}{1+r_t}\pi_{t+1}\left(1+\pi_{t+1}\right)\frac{Y_{t+1}}{Y_t}\right]$$

where $\eta_t^P = \rho^P \eta_{t-1}^P + \sigma^P \epsilon_t^P$ is a price-markup shock with $\epsilon_t^P \overset{i.i.d.}{\sim} N(0,1)$. This is Equation (6) in the main text.

A.4 Intermediate good firms

Let me start ignoring the fringe of competitors. Intermediate good firms solve:

$$\min_{L_t^j, K_{t-1}^j} w_t L_t^j + R_t^K K_{t-1}^j \tag{32}$$

$$\text{s.t. } z_t\left(L_t^j\right)^{1-\alpha}\left(K_{t-1}^j\right)^{\alpha} \geq \bar{Y_m}$$

where MC_t^j is the Lagrange multiplier associated to this problem. First order conditions are:

$$w_t = MC_t^j\left(1-\alpha\right) z_t\left(L_t^j\right)^{-\alpha}\left(K_{t-1}^j\right) \tag{33}$$

$$R_t^k = MC_t^j \alpha z_t\left(L_t^j\right)^{1-\alpha}\left(K_{t-1}^j\right) \tag{34}$$

Substituting (33) and (34) into (32) yields

$$w_t L_t^j + R_t^K K_{t-1}^j = MC_t^j Y_{mt}^j$$

which allows to rewrite the problem as a maximization of firms' profits:

$$d^j_{mt} \equiv \max_{P^j_{mt}, Y^j_{mt}} \frac{P^j_{mt}}{P_t} Y^j_{mt} - MC^j_t Y^j_{mt} \tag{35}$$

$$\text{s.t. } Y^j_{mt} = \left(\frac{P^j_{mt}}{P_t} \right)^{-\frac{\theta}{\theta-1}} Y_{mt}$$

whose FOC is

$$\frac{P^j_{mt}}{P_t} = \theta MC^j_t. \tag{36}$$

The parameter θ is the *unconstrained* markup the leader would apply in the absence of the threat of entry from the fringe of competitors. However, if one takes into account such a threat, the pricing rule becomes:

$$\frac{P^j_{mt}}{P_t} = \varpi MC^j_t, \text{ where } \varpi \equiv \min \left(\iota^{1-\alpha}, \theta \right) \tag{37}$$

where $\iota^{1-\alpha}$ is the highest markup the leader can charge without losing the market to its competitors.

Using $MC^j_t = \frac{1}{\varpi} \frac{P^j_{mt}}{P_t}$ and the intermediate good producers production function, I can rewrite (33) and (34) as, respectively:

$$w_t = \frac{1}{\varpi} \frac{P^j_{mt}}{P_t} (1-\alpha) \frac{Y^j_{mt}}{L^j_t}, \tag{38}$$

$$R^k_t = \frac{1}{\varpi} \frac{P^j_{mt}}{P_t} \alpha \frac{Y^j_{mt}}{K^j_t}. \tag{39}$$

which are equations 8 and 9 in the main text.

Let me define aggregate hours and capital as, respectively,

$$L_t \equiv \int_0^{N_{t-1}} L^j_t dj = N_{t-1} L^j_t,$$

$$K_t \equiv \int_0^{N_{t-1}} K^j_t dj = N_{t-1} K^j_t,$$

I can then apply the CES aggregator to equations (38) and (39) together with the marginal cost definition (28) and intermediate goods quantity definition (27) to get

$$w_t = \frac{1}{\varpi}(1-\alpha)\, MC_t \frac{Y_t}{L_t}, \tag{40}$$

$$R_t^k = \frac{1}{\varpi}\alpha MC_t \frac{Y_t}{K_t}. \tag{41}$$

Intermediate goods profits instead could be written as

$$\begin{aligned} d_{mt}^j &= \frac{P_{mt}^j}{P_t} Y_{mt}^j - MC_t^j Y_{mt}^j \\ &= \left(\frac{\varpi - 1}{\varpi}\right) \frac{P_{mt}^j}{P_t} Y_{mt}^j \\ &= \left(\frac{\varpi - 1}{\varpi}\right) MC_t N_t^{\theta-1} Y_{mt} \end{aligned}$$

which is Equation (10) in the main text.

A.5 Capital good firms

Capital-good firms face quadratic adjustment costs $\varphi^K(K_t, K_{t-1}) = \frac{1}{2\delta\epsilon^I}\left(\frac{K_t}{g_Y K_{t-1}} - 1\right)^2$ with derivatives

$$\varphi_{K_{t-1}}^K(K_t, K_{t-1}) = \frac{1}{\delta\epsilon^I}\left(\frac{K_t}{g_Y K_{t-1}} - 1\right)\frac{K_t}{g_Y K_{t-1}^2},$$

$$\varphi_{K_t}^K(K_t, K_{t-1}) = \frac{1}{\delta\epsilon^I}\left(\frac{K_t}{g_Y K_{t-1}} - 1\right)\frac{1}{g_Y K_{t-1}}.$$

The problem of the capital good firm is

$$J_t^K(K_{t-1}) = \max_{K_t}\left\{R_t^K K_{t-1} - \Big(K_t - (1-\delta) K_{t-1} + \varphi^K(K_t, K_{t-1}) K_{t-1}\Big) + \mathbb{E}_t \frac{J_{t+1}(P_t^o)}{1+r_t}\right\}$$

whose first order condition with respect to K_t is

$$1 + \varphi_{K_t}(K_t, K_{t-1}) K_{t-1} = \frac{1}{1+r_t}\frac{\partial J_{t+1}(P_t^o)}{\partial K_t} \equiv Q_t$$

and the envelope condition is

$$\frac{\partial J_t\left(P_t^o\right)}{\partial K_{t-1}} = R_t^K + (1-\delta) - \varphi_{K_{t-1}}^K\left(K_t, K_{t-1}\right) K_{t-1} - \varphi^K\left(K_t, K_{t-1}\right).$$

The first equation can be rewritten as

$$\frac{1}{\delta \epsilon^I}\left(\frac{K_t}{g_Y K_{t-1}} - 1\right)\frac{1}{g_Y} = Q_t - 1$$

while the second one, using $Q_t - 1 = \varphi_{K_t}\left(K_t, K_{t-1}\right) K_{t-1}$, as

$$\left(1 + r_{t-1} + \eta_t^I\right) Q_{t-1} = R_t^K + (1-\delta) + (Q_t - 1)\frac{K_t}{g_Y K_{t-1}} - \frac{1}{2\delta\epsilon^I}\left(\frac{K_t}{g_Y K_{t-1}} - 1\right)^2 + \frac{K_t}{g_Y K_{t-1}} Q_t.$$

The last two expressions are, respectively, equations (11) and (12) in the main text.

A.6 Aggregate production function

The final output aggregator implies

$$Y_t = \left(\int_0^1 \left(Y_t^o\right)^{\frac{1}{\mu^P}} do\right)^{\mu^P} = Y_t^o = Y_{mt}^o = \left(\int_0^{N_{t-1}} \left(Y_{mt}^j\right)^{\frac{1}{\theta}} dj\right)^{\theta} = N_{t-1}^{\theta} Y_{mt}^j.$$

Using the intermediate goods firms production function together with the definitions of aggregate hours and capital, the above can be rewritten as

$$Y_t = z_t N_t^{\theta-1} K_t^{\alpha} L_t^{1-\alpha}$$

which is Equation (19) in the main text.

A.7 Summary of the baseline model

The equilibrium system of equations characterizing the model is:
Households:

$$C_t \equiv \int_i c_t\left(e_t^i, a_{t-1}^i\right) di$$

$$A_t \equiv \int_i a_t \left(e_t^i, a_{t-1}^i\right) di$$

$$\pi_t^W \left(1+\pi_t^W\right) = \eta_t^W + \kappa^W \left[\gamma L_t^{1+\varrho} - \frac{\epsilon-1}{\epsilon} \left(1-\tau_t\right) w_t L_t u' \left(C_t^*\right)\right] + \beta \nu_t \mathbb{E}_t \pi_{t+1}^W \left(1+\pi_{t+1}^W\right)$$

$$C_t^* \equiv \left(\int_i \frac{e_t^i}{\int e_t^i di} \left(c_t^i\right)^{-1} di\right)^{-1}$$

Final goods firms:

$$\pi_t \left(1+\pi_t\right) = \eta_t^P + \kappa^P \left(\mu^P MC_t - 1\right) + \mathbb{E}_t \frac{1}{1+r_t} \pi_{t+1} \left(1+\pi_{t+1}\right) \frac{Y_{t+1}}{Y_t}$$

Intermediate goods firms:

$$\alpha MC_t \frac{Y_t}{K_{t-1}} = \varpi R_t^K \tag{42}$$

$$\left(1-\alpha\right) MC_t \frac{Y_t}{L_t} = \varpi w_t \tag{43}$$

Capital goods firms:

$$I_t = K_t + \eta_t^I \frac{1}{2\delta\epsilon^I} \left(\frac{K_t}{g_Y K_{t-1}} - 1\right)^2 K_{t-1} - \left(1-\delta\right) K_{t-1}$$

$$\frac{1}{g_Y \delta \epsilon^I} \left(\frac{K_t}{g_Y K_{t-1}} - 1\right) \eta_t^I = Q_t - 1$$

$$\left(1 + r_{t-1} + \eta_t^I\right) Q_{t-1} = R_t^K - \left(\frac{K_t}{g_Y K_{t-1}} - \left(1-\delta\right)\right) - \frac{1}{2\delta\epsilon^I} \left(\frac{K_t}{g_Y K_{t-1}} - 1\right)^2 + \frac{K_t}{g_Y K_{t-1}} Q_t$$

Research and development firms:

$$N_t = \chi_t N_{t-1} S_t^{\rho_N} K_{t-1}^{-\rho_N} + \left(1-\phi\right) N_{t-1} - \frac{1}{\phi \epsilon^N} \left(\frac{N_t}{g_N N_{t-1}} - 1\right) N_{t-1} \tag{44}$$

$$V_t^N = \left(\frac{\varpi - 1}{\varpi}\right) MC_t Y_t + \left(1-\phi\right) \mathbb{E}_t \left[\frac{1}{1+r_t} V_{t+1}^N \frac{N_{t-1}}{N_t}\right] \tag{45}$$

$$\chi_t \frac{S_t^{\rho_N - 1}}{K_{t-1}^{\rho_N}} \left(1-\phi\right) \mathbb{E}_t \left[\frac{1}{1+r_t} V_{t+1}^N \frac{N_{t-1}}{N_t}\right] = 1 \tag{46}$$

Final output and aggregated firms profits:

$$Y_t = z_t N_{t-1}^{\theta-1} K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{47}$$

$$D_t = Y_t - w_t L_t - S_t - I_t - \frac{1}{2\kappa^P \left(\mu^P - 1\right)} \pi_t^2 Y_t$$

Fiscal policy:

$$\tau_t w_t L_t = \left(R_{t-1} - g_Y\right) g_Y^t B + G_t \tag{48}$$

Monetary policy and inflation:

$$i_t = r^* + \phi_\pi \pi_t + \eta_t^{mp}$$

$$1 + r_{t-1} = \frac{1 + i_{t-1}}{1 + \pi_t}$$

$$\pi_t^W = \pi_t \frac{w_t}{w_{t-1}}$$

Market clearing:

$$A_t = g_Y^{t+1} B \tag{49}$$

Exogenous processes:

$$\begin{aligned}
z_t &= \left(1 - \rho^z\right) \bar{z} + \rho^z z_{t-1} + \sigma^z \epsilon_t^z \\
\chi_t &= \left(1 - \rho^\chi\right) \bar{\chi} + \rho^\chi \chi_{t-1} + \sigma^\chi \epsilon_t^\chi \\
\eta_t^{mp} &= \rho^{mp} \eta_{t-1}^{mp} + \sigma^{mp} \epsilon_t^{mp} \\
\eta_t^I &= \rho^I \eta_{t-1}^I + \sigma^I \epsilon_t^I \\
\nu_t &= \rho^\nu \nu_{t-1} + \sigma^\nu \epsilon_t^\nu \\
\frac{G_t}{g_Y^t} &= \left(1 - \rho^G\right) \bar{G} + \rho^G \frac{G_{t-1}}{g_Y^{t-1}} + \sigma^G \epsilon_t^G \\
\eta_t^P &= \rho^P \eta_{t-1}^P + \sigma^P \epsilon_t^P \\
\eta_t^W &= \rho^W \eta_{t-1}^W + \sigma^W \epsilon_t^W
\end{aligned} \tag{50}$$

where $V_t^N \equiv V_t N_t$.

A.8 Summary of the baseline stationary model

The economy described in Section 2 and its system of equation in A.7 is non-stationary and, as such, does not allow the employment Sequence Space Jacobian techniques for its solution and simulation. Production function (19) shows that the growth rate of the economy along the balanced growth path is $g_Y \equiv \frac{\theta-1}{1-\alpha} g_N$. In that respect, let me define, for generic jump $\mathfrak{j}_t$ and state $\mathfrak{s}_t$ variables, their stationary counterpart respectively as $\tilde{\mathfrak{j}}_t \equiv \frac{\mathfrak{j}_t}{g_Y^t}$ and $\tilde{\mathfrak{s}}_t \equiv \frac{\mathfrak{s}_t}{g_Y^{t+1}}$. I detrend all jump and state variables according to that, except for varieties, of which the stationary counterpart is defined as $\tilde{N}_t \equiv \frac{N_t}{g_N^{t+1}}$.

The equilibrium system of equations characterizing the BGP stationary model is: **Households:**

$$\tilde{C}_t \equiv \int_i \tilde{c}_t \left(e_t^i, \tilde{a}_{t-1}^i\right) di$$

$$\tilde{A}_t \equiv \int_i \tilde{a}_t \left(e_t^i, \tilde{a}_{t-1}^i\right) di$$

$$\pi_t^W \left(\pi_t^W - 1\right) = \eta_t^W + \kappa^W \left[\gamma L_t^{1+\varrho} - \frac{\epsilon - 1}{\epsilon} \left(1 - \tau_t\right) \tilde{w}_t L_t u' \left(\tilde{C}_t^*\right)\right] + \beta \nu_t \pi_{t+1}^W \left(\pi_{t+1}^W - 1\right)$$

$$\tilde{C}_t^* \equiv \left(\int_i \frac{e_t^i}{\int e_t^i di} \left(\tilde{c}_t^i\right)^{-1} di\right)^{-1}$$

Final goods firms:

$$\pi_t \left(\pi_t - 1\right) = \kappa^P \left(\mu_t^P MC_t - 1\right) + \mathbb{E}_t \frac{1}{1 + r_t} \pi_{t+1} \left(\pi_{t+1} - 1\right) \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} g_Y$$

Intermediate goods firms:

$$\alpha MC_t \frac{\tilde{Y}_t}{\tilde{K}_{t-1}} = \varpi R_t^K \tag{51}$$

$$\left(1 - \alpha\right) MC_t \frac{\tilde{Y}_t}{L_t} = \varpi \tilde{w}_t \tag{52}$$

Capital goods firms:

$$\tilde{I}_t = \tilde{K}_t g_Y + \frac{1}{2\delta\epsilon^I} \left(\frac{\eta_t^I \tilde{K}_t}{\tilde{K}_{t-1}} - 1\right)^2 \tilde{K}_{t-1} - \left(1 - \delta\right) \tilde{K}_{t-1}$$

$$\frac{1}{\delta\epsilon^I g_Y} \left(\frac{\eta_t^I \tilde{K}_t}{\tilde{K}_{t-1}} - 1\right) = Q_t - 1$$

$$\left(1+r_{t-1}+\eta_t^I\right)=R_t^K-\left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}}-(1-\delta)\right)-\frac{1}{2\delta\epsilon^I}\left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}}-1\right)^2+\frac{\tilde{K}_t}{\tilde{K}_{t-1}}Q_t$$

Research and development firms:

$$g_N\tilde{N}_t=\chi_t\tilde{N}_{t-1}\tilde{S}_t^{\rho_N}\tilde{K}_{t-1}^{-\rho_N}+(1-\phi)\,\tilde{N}_{t-1}-\frac{1}{\phi\epsilon^N}\left(\frac{\tilde{N}_t}{\tilde{N}_{t-1}}-1\right)\tilde{N}_{t-1} \tag{53}$$

$$\tilde{V}_t^N=\left(\frac{\varpi-1}{\varpi}\right)MC_t\tilde{Y}_t+(1-\phi)\,\mathbb{E}_t\left[\frac{g_Y}{\varsigma_tR_t}\tilde{V}_{t+1}^N\frac{N_{t-1}}{g_NN_t}\right] \tag{54}$$

$$\chi_t\frac{\tilde{S}_t^{\rho_N-1}}{\tilde{K}_{t-1}^{\rho_N}}\,(1-\phi)\,\mathbb{E}_t\left[\frac{g_Y}{\varsigma_tR_t}\tilde{V}_{t+1}^N\frac{\tilde{N}_{t-1}}{g_N\tilde{N}_t}\right]=1 \tag{55}$$

Final output and aggregated firms profits:

$$\tilde{Y}_t=z_t\tilde{N}_{t-1}^{\theta-1}\tilde{K}_{t-1}^{\alpha}L_t^{1-\alpha} \tag{56}$$

$$\tilde{D}_t=\tilde{Y}_t-\tilde{w}_tL_t-\tilde{S}_t-\tilde{I}_t-\frac{1}{2\kappa^P\left(\mu^P-1\right)}\left(\pi_t-1\right)^2\tilde{Y}_t$$

Fiscal policy:

$$\tau_t\tilde{w}_tL_t=\left(\varsigma_{t-1}R_{t-1}-g_y\right)B+\tilde{G}_t \tag{57}$$

Monetary policy and inflation: Taylor 1

$$i_t=r^*+\phi_\pi\pi_t+\eta_t^{mp}$$

$$1+r_{t-1}=\frac{1+i_{t-1}}{1+\pi_t}$$

$$\pi_t^W=\pi_t\frac{\tilde{w}_t}{\tilde{w}_{t-1}}g_Y$$

Market clearing:

$$\tilde{A}_t=B \tag{58}$$

Exogenous processes:

$$\begin{aligned}z_t&=(1-\rho^z)\,\bar{z}+\rho^zz_{t-1}+\sigma^z\epsilon_t^z\\ \chi_t&=(1-\rho^\chi)\,\bar{\chi}+\rho^\chi\chi_{t-1}+\sigma^\chi\epsilon_t^\chi\\ \eta_t^{mp}&=\rho^{mp}\eta_{t-1}^{mp}+\sigma^{mp}\epsilon_t^{mp}\end{aligned} \tag{59}$$

$$\eta_t^I = \rho^I \eta_{t-1}^I + \sigma^I \epsilon_t^I$$

$$\nu_t = \rho^\nu \nu_{t-1} + \sigma^\nu \epsilon_t^\nu$$

$$\tilde{G}_t = \left(1 - \rho^G\right) \bar{G} + \rho^G \tilde{G}_{t-1} + \sigma^G \epsilon_t^G$$

$$\eta_t^P = \rho^P \eta_{t-1}^P + \sigma^P \epsilon_t^P$$

$$\eta_t^W = \rho^W \eta_{t-1}^W + \sigma^W \epsilon_t^W$$

A.9 Representative agent model

The equilibrium system of equations characterizing the model is:
Households:

$$C_t + A_t = (1 - \tau_t) w_t L_t + R_{t-1} A_{t-1} + D_t$$

$$C_t^{-1} = \beta \mathbb{E}_t \left[C_{t+1}^{-1} (1 + r_t)\right] \tag{60}$$

$$\pi_t^W \left(1 + \pi_t^W\right) = \eta_t^W + \kappa^W \left[\gamma L_t^{1+\varrho} - \frac{\epsilon - 1}{\epsilon} (1 - \tau_t) w_t L_t C_t^{-1}\right] + \beta \nu_t \mathbb{E}_t \pi_{t+1}^W \left(1 + \pi_{t+1}^W\right)$$

Final goods firms:

$$\pi_t (1 + \pi_t) = \eta_t^P + \kappa^P \left(\mu^P MC_t - 1\right) + \mathbb{E}_t \frac{1}{1 + r_t} \pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t}$$

Intermediate goods firms:

$$\alpha MC_t \frac{Y_t}{K_{t-1}} = \varpi R_t^K \tag{61}$$

$$(1 - \alpha) MC_t \frac{Y_t}{L_t} = \varpi w_t \tag{62}$$

Capital goods firms:

$$I_t = K_t + \eta_t^I \frac{1}{2\delta \epsilon^I} \left(\frac{K_t}{g_Y K_{t-1}} - 1\right)^2 K_{t-1} - (1 - \delta) K_{t-1}$$

$$\frac{1}{g_Y \delta \epsilon^I} \left(\frac{K_t}{g_Y K_{t-1}} - 1\right) \eta_t^I = Q_t - 1$$

$$\left(1+r_{t-1}+\eta_t^I\right)Q_{t-1}=R_t^K-\left(\frac{K_t}{g_Y K_{t-1}}-(1-\delta)\right)-\frac{1}{2\delta\epsilon^I}\left(\frac{K_t}{g_Y K_{t-1}}-1\right)^2+\frac{K_t}{g_Y K_{t-1}}Q_t$$

Research and development firms:

$$N_t=\chi_t N_{t-1}S_t^{\rho_N}K_{t-1}^{-\rho_N}+(1-\phi)N_{t-1}-\frac{1}{\phi\epsilon^N}\left(\frac{N_t}{g_N N_{t-1}}-1\right)N_{t-1} \tag{63}$$

$$V_t^N=\left(\frac{\varpi-1}{\varpi}\right)MC_tY_t+(1-\phi)\mathbb{E}_t\left[\frac{1}{R_t}V_{t+1}^N\frac{N_{t-1}}{N_t}\right] \tag{64}$$

$$\chi_t\frac{S_t^{\rho_N-1}}{K_{t-1}^{\rho_N}}(1-\phi)\mathbb{E}_t\left[\frac{1}{R_t}V_{t+1}^N\frac{N_{t-1}}{N_t}\right]=1 \tag{65}$$

Final output and aggregated firms sector profits:

$$Y_t=z_tN_{t-1}^{\theta-1}K_{t-1}^{\alpha}L_t^{1-\alpha} \tag{66}$$

$$D_t=Y_t-w_tL_t-S_t-I_t-\frac{1}{2\kappa^P\left(\mu^P-1\right)}\pi_t^2Y_t$$

Fiscal policy:

$$\tau_tw_tL_t=\left(R_{t-1}-g_Y\right)g_Y^tB+G_t \tag{67}$$

Monetary policy and inflation:

$$i_t=r^*+\phi_\pi\pi_t+\eta_t^{mp}$$

$$1+r_{t-1}=\frac{1+i_{t-1}}{1+\pi_t}$$

$$\pi_t^W=\pi_t\frac{w_t}{w_{t-1}}$$

Market clearing:

$$A_t=g_Y^{t+1}B \tag{68}$$

Exogenous processes:

$$z_t=(1-\rho^z)\bar{z}+\rho^zz_{t-1}+\sigma^z\epsilon_t^z$$

$$\chi_t=(1-\rho^\chi)\bar{\chi}+\rho^\chi\chi_{t-1}+\sigma^\chi\epsilon_t^\chi \tag{69}$$

$$\eta_t^{mp}=\rho^{mp}\eta_{t-1}^{mp}+\sigma^{mp}\epsilon_t^{mp}$$

$$\eta_t^I = \rho^I \eta_{t-1}^I + \sigma^I \epsilon_t^I$$

$$\nu_t = \rho^\nu \nu_{t-1} + \sigma^\nu \epsilon_t^\nu$$

$$\frac{G_t}{g_Y^t} = \left(1-\rho^G\right) \bar{G} + \rho^G \frac{G_{t-1}}{g_Y^{t-1}} + \sigma^G \epsilon_t^G$$

$$\eta_t^P = \rho^P \eta_{t-1}^P + \sigma^P \epsilon_t^P$$

$$\eta_t^W = \rho^W \eta_{t-1}^W + \sigma^W \epsilon_t^W$$

A.10 Representative agent model: stationary

The equilibrium system of equations characterizing the stationary model is:
Households:

$$\tilde{C}_t + \tilde{A}_t g_Y = (1-\tau_t)\,\tilde{w}_t L_t + \varsigma_{t-1} R_{t-1} \tilde{A}_{t-1} + \tilde{D}_t$$

$$\tilde{C}_t^{-1} = \beta \mathbb{E}_t \left[\tilde{C}_{t+1}^{-1} \frac{1+r_t}{g_Y} \right] \tag{70}$$

$$\pi_t^W \left(1+\pi_t^W\right) = \eta_t^W + \kappa^W \left[\gamma L_t^{1+\varrho} - \frac{\epsilon-1}{\epsilon} (1-\tau_t)\,\tilde{w}_t L_t \tilde{C}_t^{-1} \right] + \beta \nu_t \pi_{t+1}^W \left(1+\pi_{t+1}^W\right)$$

Final goods firms:

$$\pi_t\left(\pi_t - 1\right) = \kappa^P \left(\mu_t^P MC_t - 1\right) + \frac{1}{1+r_t} \pi_{t+1}\left(\pi_{t+1}-1\right) \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} g_Y$$

Intermediate goods firms:

$$\alpha MC_t \frac{\tilde{Y}_t}{\tilde{K}_{t-1}} = \varpi R_t^K \tag{71}$$

$$(1-\alpha)\, MC_t \frac{\tilde{Y}_t}{L_t} = \varpi \tilde{w}_t \tag{72}$$

Capital goods firms:

$$\tilde{I}_t = \tilde{K}_t g_Y + \frac{1}{2\delta\epsilon^I} \left(\frac{\eta_t^I \tilde{K}_t}{\tilde{K}_{t-1}} - 1 \right)^2 \tilde{K}_{t-1} - (1-\delta)\,\tilde{K}_{t-1}$$

$$\frac{1}{\delta \epsilon^I g_Y} \left(\frac{\eta_t^I \tilde{K}_t}{\tilde{K}_{t-1}} - 1 \right) = Q_t - 1$$

$$\left(1 + r_{t-1} + \eta_t^I\right) = R_t^K - \left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}} - (1-\delta)\right) - \frac{1}{2\delta\epsilon^I}\left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}} - 1\right)^2 + \frac{\tilde{K}_t}{\tilde{K}_{t-1}} Q_t$$

Research and development firms:

$$g_N \tilde{N}_t = \chi_t \tilde{N}_{t-1} \tilde{S}_t^{\rho_N} \tilde{K}_{t-1}^{-\rho_N} + (1-\phi)\,\tilde{N}_{t-1} - \frac{1}{\phi\epsilon^N}\left(\frac{\tilde{N}_t}{\tilde{N}_{t-1}} - 1\right)\tilde{N}_{t-1} \tag{73}$$

$$\tilde{V}_t^N = \left(\frac{\varpi - 1}{\varpi}\right) MC_t \tilde{Y}_t + (1-\phi)\,\mathbb{E}_t\left[\frac{g_Y}{1+r_t}\tilde{V}_{t+1}^N \frac{N_{t-1}}{g_N N_t}\right] \tag{74}$$

$$\chi_t \frac{\tilde{S}_t^{\rho_N - 1}}{\tilde{K}_{t-1}^{\rho_N}}\,(1-\phi)\,\mathbb{E}_t\left[\frac{g_Y}{1+r_t}\tilde{V}_{t+1}^N \frac{\tilde{N}_{t-1}}{g_N \tilde{N}_t}\right] = 1 \tag{75}$$

Final output and aggregated firms sector profits:

$$\tilde{Y}_t = z_t \tilde{N}_{t-1}^{\theta-1} \tilde{K}_{t-1}^{\alpha} L_t^{1-\alpha} \tag{76}$$

$$\tilde{D}_t = \tilde{Y}_t - \tilde{w}_t L_t - \tilde{S}_t - \tilde{I}_t - \frac{1}{2\kappa^P\left(\mu^P - 1\right)}\left(\pi_t - 1\right)^2 \tilde{Y}_t$$

Fiscal policy:

$$\tau_t \tilde{w}_t L_t = \left(\varsigma_{t-1} R_{t-1} - g_y\right) B + \tilde{G}_t \tag{77}$$

Monetary policy and inflation:

$$i_t = r^* + \phi_\pi \pi_t + \eta_t^{mp}$$

$$1 + r_{t-1} = \frac{1 + i_{t-1}}{1 + \pi_t}$$

$$\pi_t^W = \pi_t \frac{\tilde{w}_t}{\tilde{w}_{t-1}} g_Y$$

Market clearing:

$$\tilde{A}_t = B \tag{78}$$

Extra:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t + \tilde{S}_t + \frac{1}{2\kappa^P\left(\mu^P - 1\right)}\pi_t^2 \tilde{Y}_t$$

Exogenous processes:

$$z_t = \left(1 - \rho^z\right)\bar{z} + \rho^z z_{t-1} + \sigma^z \epsilon_t^z$$

$$\chi_t = (1-\rho^{\chi})\,\bar{\chi} + \rho^{\chi}\chi_{t-1} + \sigma^{\chi}\epsilon_t^{\chi} \tag{79}$$

$$\eta_t^{mp} = \rho^{mp}\eta_{t-1}^{mp} + \sigma^{mp}\epsilon_t^{mp}$$

$$\eta_t^{I} = \rho^{I}\eta_{t-1}^{I} + \sigma^{I}\epsilon_t^{I}$$

$$\nu_t = \rho^{\nu}\nu_{t-1} + \sigma^{\nu}\epsilon_t^{\nu}$$

$$\tilde{G}_t = \left(1-\rho^{G}\right)\bar{G} + \rho^{G}\tilde{G}_{t-1} + \sigma^{G}\epsilon_t^{G}$$

$$\eta_t^{P} = \rho^{P}\eta_{t-1}^{P} + \sigma^{P}\epsilon_t^{P}$$

$$\eta_t^{W} = \rho^{W}\eta_{t-1}^{W} + \sigma^{W}\epsilon_t^{W}$$

A.11 Two-agents model

Defining C_t^{HtM} as consumption from hand-to-mouth households and C_t^{PI} as consumption from permanent income consumers, the equilibrium system of equation characterizing the two-agents model is the same as for the representative agent model (Section A.9) with the households block changed to:

$$C_t + A_t = (1-\tau_t)\,w_t L_t + R_{t-1}A_{t-1} + D_t$$

$$C_t^{HtM} = (1-\tau_t)\,w_t L_t$$

$$C_t = \lambda C_t^{HtM} + (1-\lambda)\,C_t^{PI}$$

$$\left(C_t^{PI}\right)^{-1} = \beta\mathbb{E}_t\left[\left(C_{t+1}^{PI}\right)^{-1}(1+r_t)\right] \tag{80}$$

B Additional Figures

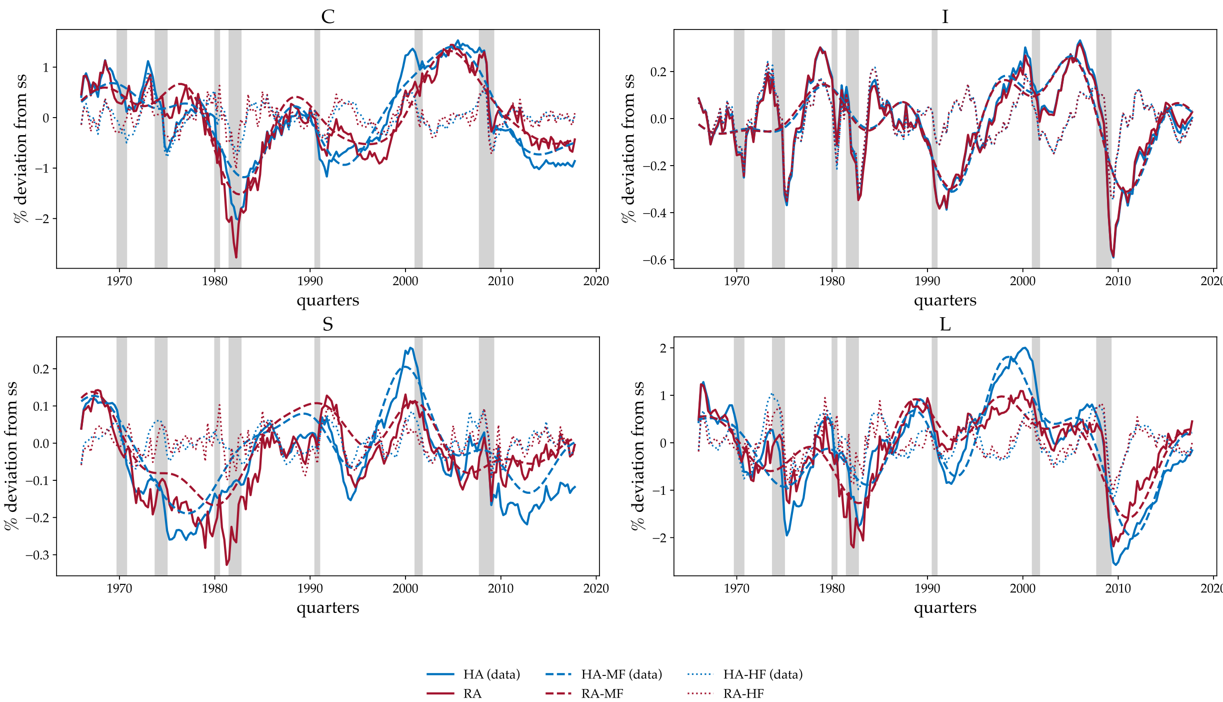

Figure 5: Simulated model's (HA in blue and RA in red) time series at the medium-term business cycle frequency (MT, fluctuations within 2-200 quarters, solid lines), medium-frequency (MF, fluctuations within 32-200 quarters, dashed lines) and high-frequency (HF, fluctuations within 2-32 quarters, dotted lines), for consumption C, investment I, research and development S, and hours worked L, extracted with the Christiano and Fitzgerald (2003)'s bandpass filter. Grey bars denote NBER recessions. Time period is 1966Q1 to 2017Q4.

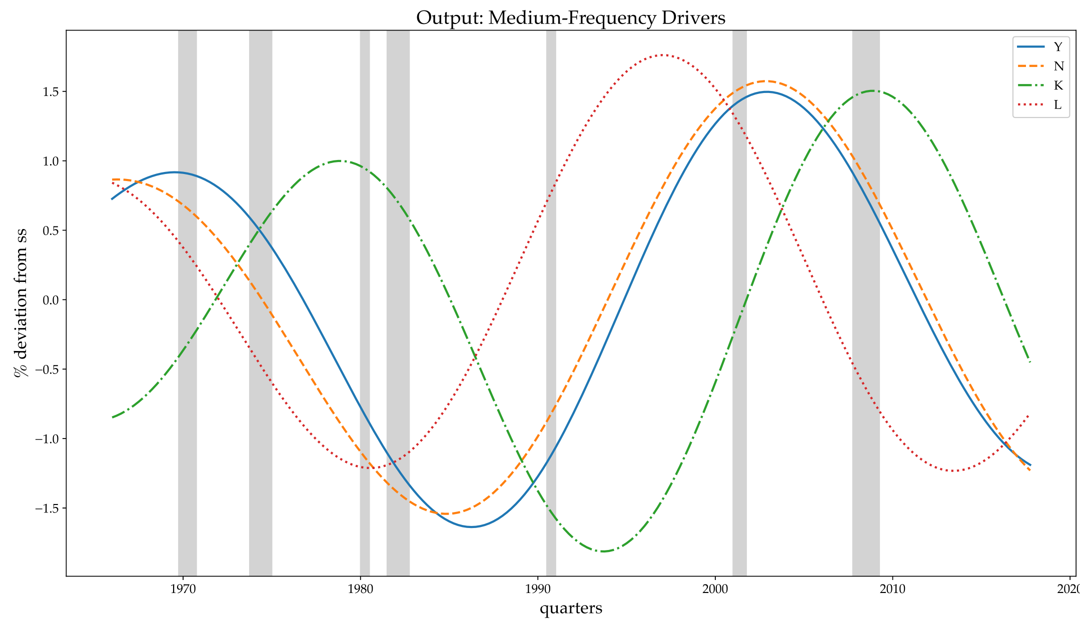

Figure 6: Low frequency cycles in the baseline model for Output Y, Varieties N, Capital K and Labor L. Grey bars denote NBER recessions. Time period is 1966Q1 to 2017Q4.

C Data appendix

C.1 Data used in Bayesian estimation

In this section I describe the data used for estimating the heterogeneous agents model described in Section 2. I have used eight observables for the U.S. economy, for the period 1964Q1-2019Q4, which are downloadable from the FRED database of the Federal Reserve Bank of St. Louis. They are (identifier in parentheses): nominal GDP (GDP), nominal investment (GDPI), nominal personal consumption expenditures (PCE), Nominal Research and development (Y006RC1Q027SBEA), Average hourly earnings of production employees: private (AHETPI), Nonfarm Business Sector: Hours of All Persons (HOANBS), Fed Funds rate (FEDFUNDS) and personal consumption expenditures deflator excluding food and energy, index 2012 (PCEPILFE). All series are aggregated to quarterly frequency. Output, investment, consumption, R&D and wages are deflated by the GDP deflator index 2012 (GDPDEF).

All series but inflation and interest rate are logged, multiplied by 100 and bandpassed at the medium-term (2, 200) with the Christiano and Fitzgerald (2003)'s bandpass filter. Inflation and interest rate are demeaned. I drop the first and last two years of observations since are poorly estimated by the bandpass.

C.2 Data used in empirical analysis

In this section I describe the data used in the empirical analysis in Section 5. National accounts data are from the World Development Indicators of the World Bank. The variables I use are GDP per capita (constant LCU), Gross capital formation (% of GDP), General government final consumption expenditure (% of GDP), Imports of goods and services (% of GDP), Exports of goods and services (% of GDP), Population, total, Households and NPISHs final consumption expenditure (% of GDP), GDP per capita, PPP (constant 2021 international $), Net barter terms of trade index (2015 = 100), Manufacturing, value added (% of GDP), Research and development expenditure (% of GDP) and GDP (constant 2015 US$). Trade openness is defined as the share of exports plus imports in GDP. The panel component starts at 1960 and ends in 2022, however data availability across time is different from country to country. The inequality data are from the World Inequality Database. I focus on the Gini index of pre-tax income countries's distribution. This dataset is available from the 1980 until 2023, with differences across countries. I spline missing values and drop countries which do not have at least 40 years of observations, for which the estimation of the medium-term business cycle would be poor. The volatilities used in regression (24) are computed over the medium-term frequency of the variable of interest, filtered with the Christiano and Fitzgerald (2003)'s bandpass filter.

D Estimation results

Estimated shock volatilities and autoregressive coefficients are reported in Table 4. The closest estimated model in the literature to mine is the two-asset heterogeneous agents model in Auclert et al. (2021), even though the latter is estimated on high-frequency data while mine on medium-term ones. The volatility of the monetary policy shock (0.12) is in line with the one found in Auclert et al. (2021), however the persistence of the shock is slightly higher in my model (0.90) than in theirs (0.83). My model estimates price markup shocks to be more volatile than wage markup shocks, while the opposite holds true in Auclert et al. (2021). Also, in my model wage markup shocks are more persistent than price markup shocks, and viceversa in Auclert et al. (2021). Both features might be due to the fact that Auclert et al. (2021) focus on the case of symmetry between price and wage rigidities while I focus on the case in which wages are more rigid than prices. The preference shock behaves similarly in the two models, while I estimate the government expenditure shock to be more persistent and less volatile than Auclert et al. (2021) do. The technology shock volatility is higher than the one estimated in Auclert et al. (2021), however it is in line with previous estimates (see Smets and Wouters, 2007), while the investment shock is less volatile and more persistent than common estimates. The R&D shock is instead very persistent with a half the volatility (0.05) of the investment in physical capital shock (0.11).

Table 4: Shocks persistencies and estimated volatilities

		Prior			Posterior			
Parameter	Description	Distribution	Mean	S.D.	Mode	Mean	5%	95%
ρ^{mp}	Monetary policy shock	Beta	0.5	0.2	0.90	0.91	0.84	0.95
ρ^{P}	Price markup shock	Beta	0.5	0.2	0.30	0.30	0.24	0.35
ρ^{G}	Government expend. shock	Beta	0.5	0.2	0.93	0.92	0.86	0.97
ρ^{ν}	Discount factor shock	Beta	0.5	0.2	0.93	0.93	0.91	0.94
ρ^{z}	Productivity shock	Beta	0.5	0.2	0.95	0.94	0.91	0.97
ρ^{I}	Investment shock	Beta	0.5	0.2	0.92	0.92	0.90	0.93
ρ^{W}	Wage markup shock	Beta	0.5	0.2	0.55	0.53	0.48	0.59
ρ^{χ}	R&D shock	Beta	0.5	0.2	0.99	0.98	0.95	0.99
σ^{mp}	Monetary policy shock	Inv. gamma	0.4	4	0.12	0.13	0.11	0.15
σ^{P}	Price markup shock	Inv. gamma	0.4	4	0.49	0.49	0.45	0.54
σ^{G}	Government expend. shock	Inv. gamma	0.4	4	0.11	0.11	0.10	0.12
σ^{ν}	Discount factor shock	Inv. gamma	0.4	4	0.05	0.05	0.04	0.06
σ^{z}	Productivity shock	Inv. gamma	0.4	4	0.33	0.37	0.25	0.51
σ^{I}	Investment shock	Inv. gamma	0.4	4	0.11	0.11	0.11	0.13
σ^{W}	Wage markup shock	Inv. gamma	0.4	4	0.16	0.18	0.15	0.21
σ^{χ}	R&D shock	Inv. gamma	0.4	4	0.05	0.05	0.05	0.06

Notes: Entries refer to mode, mean and the 5 and 95 percentiles of the posterior distribution of the parameters obtained by the Metropolis Hastings algorithm with 1'000'000 draws. Acceptance rate is 0.25.