# Beyond Indication-Based Pricing: Information as a Regulatory Tool for Pharmaceuticals

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**Abstract**

First-Degree Price Discrimination achieves efficiency in monopoly settings by having the producer appropriate the total surplus, while uniform pricing leads to higher consumer surplus but is inefficient. The market for multi-indication drugs – those used to treat multiple conditions – has two features that distinguish it from a static monopoly model. Firstly, the monopoly is time-bound and is followed by a competitive market. Secondly, manufacturers have private information about which groups of patients their drug could be useful for and must incur a cost to reveal each one. I extend Bergemann et al. (2015) to this setting to develop a mechanism which maximises consumer surplus and achieves weakly higher efficiency than First-Degree Price Discrimination (which can be inefficient as the monopolist does not internalise the long-term benefit). By conditioning market segmentation on the manufacturer's revealed information, a regulator can incentivise it to conduct costly trials for the most socially beneficial uses of their drugs and maximise achievable total and consumer surplus. Ex-post, First-Degree Price Discrimination may appear as a specific case as a result of the segmentation - as such, I reveal conditions under which First-Degree Price Discrimination may be efficient. I also provide solutions for achieving market segmentation that minimizes variance in prices or individual consumer surplus while still achieving the primary aims of maximising achievable surplus. In doing so I make the case for regulators to use market segmentation as a tool to provide incentives for manufacturers.

# 1 Introduction

Expenditure on medicine across the globe went up from $887 billion in 2010 to $1.48 trillion in 2022 (IQVIA Institute, 2023), yet access to pharmaceutical drugs still remains an issue. About 30% of Americans reported not taking prescribed medication because they can't afford it (Kirzinger et al., 2019), while in countries with state insurers, similar issues equate to the denial of access to certain pharmaceuticals because of economic infeasibility. Given that production costs after the launch of a drug are low (Lakdawalla, 2018), access issues might point to market inefficiencies. These inefficiencies can be critical in the case of the rapidly growing range of drugs with multiple uses, known as multi-indication drugs where whole patient groups can find themselves priced out of the market. Furthermore, the growing field of drug re-purposing – that systemically finds potential uses for existing drugs – can be hampered if research is not appropriately linked to returns for manufacturers (Pushpakom et al. (2019),van der Pol et al. (2023)).

Manufacturers of new drugs initially have a monopoly on its sale before the patent period ends and competitors are allowed to enter the market. Their inability to price discriminate during in the short run or internalise the societal benefits from the long-run leads to inefficiencies. Being best placed to know potential uses of their drugs, manufacturer will not conduct costly trials unless they can recoup their investment through sales in the short run; patients who would be priced out or who suffer from rare conditions may never have trials conducted for their diseases. Indication-based pricing is a proposal under consideration in several countries (Preckler and Espin, 2022) which allows manufacturers to charge prices for patients based on the condition being treated (referred to as 'indication') however, it suffers from two drawbacks – in the short run it may lead to increased prices, leaving very little surplus for patients and payers (Goodman (2023),Chandra and Garthwaite (2017), Jiang et al. (2024)) while the long-run it is not socially efficient as manufacturers optimise over the monopoly period (van der Pol et al., 2023). The first point implies that manufacturers are receiving too much information about individual patients — in this paper I structure manufacturers' access to information in a way that mitigates both these issues.

In this paper I design a mechanism wherein a regulator can induce the manufacturer of a multi-indication drug to bear the cost of revealing private information about uses of the drug in order to maximise total surplus by maximising consumer surplus. This is achieved by providing the manufacturer with noisy signals about individual consumers' intended use by assigning them to segments. As such the basic mechanism is an extension and application of Bergemann et al. (2015) in two ways; I assume the monopoly is temporary, followed by an infinite period of perfect competition, and I assume the manufacturer must decide whether to sell to groups of patients (where each group is defined by its valuation) by incurring a fixed cost for each group. I also show adaptations of this mechanism that deal with potential considerations of distribution of consumer surplus among the patient population. I find that total surplus is maximised by restricting producer surplus to an optimal minimum level such that increase in consumer surplus drives the increase in efficiency.

The setup can be interpreted both as the introduction of a regulator in a free market as in the US, or as one in which a regulator is introduced into a single payer system where the payer acts on behalf of individual patients — I use the former as the base case and elabourate upon the distinction when relevant. Throughout, I assume that consumer/payer valuation of the drug is determined by patient indication. Section 2 illustrates the issues identified and how the basic mechanism works.

I show that in this setup, first-degree price discrimination is not efficient because the manufacturer does not internalise the long-term surplus generated and only makes investment decisions for the monopoly period. Since the manufacturer is the only one who can make investments, it it possible that an outcome that achieves complete efficiency is not feasible. The mechanism in this paper achieves the highest feasible total surplus and consumer surplus and, in doing so, outstrips both Indication-based pricing and uniform pricing in both dimensions. It does so by conditioning the quality of information about individuals the manufacturer receives on its investment into trials for socially efficient uses of their drugs; thereby re-investing surplus in the short-run for benefits in the long-run.

The mechanism ensures the optimal level of revelation of the manufacturer's private information. Due to the cost, full revelation is not always optimal for either party — I find that it is always in the regulator's interest to induce revelation of indications that are socially efficient (those whose cost is less than the total value to patients) while the manufacturer's optimal choice under Indication-based pricing is a subset of these; under uniform pricing it is a further subset. Maximising consumer surplus strictly requires the revelation of only the regulator's preferred set; extra revelation harms total surplus which in turn reduces consumer surplus.

The mechanism ensures that a manufacturer is never worse off for revealing the exact set of indications that the regulator desires. Deterring exclusion of desirable indications requires providing the manufacturer a return weakly better than its outside option – deterring extra revelation is then easily achieved by not compensating costs for it. I identify the outside option as the profit the manufacturer can achieve by charging a uniform price over a profit maximising subset of indications. I characterise this set of profit maximising indications as ones where the ironed increase in revenue from inclusion must be higher than ironed increase in cost of inclusion, both from higher to lower valuations and lower to higher. I present a quick algorithm to find the profit maximising subset from any larger set of indications in section 5.4. [1]

In certain cases it is possible that a regulator is concerned with increasing access to patients for rare diseases which may not be socially optimal. I show that this is also possible through the same mechanism. This result is illustrated in the example in section 2. I also provide an upper bound on this output: we know the manufacturer's outside option so we know how must consumer surplus is available for reallocation towards output.

Since segmentations that maximise consumer well-being are non-unique, I also characterise solutions that meet criteria regulators might find desirable — those that minimise variance in price for all consumers or those that minimise variance in consumer surplus for all consumers. I show that the methodology for achieving both relies heavily on the underlying population distribution among indications. Price variance minimisation relies on avoiding the creation of segments for whom the manufacturer's optimal price is high. For the case of consumer surplus

[1] This also works as a standalone solution to the question of optimal market selection when a producer must pay a fixed cost to open up new markets for a good, but must charge the same price across them.

minimisation, segmentation varies based on the relationship between different valuations. Variance in this case is minimised by prioritising placement of consumers in segments where gain in consumer surplus is low. I provide detailed answers for how to achieve both for the case of 3 indications in section 5.7.

While these results fit both scenarios of individual payers and a single payer without co-payments, I show that the results stand for setting with insured individuals with co-payments that are a fixed ratio of expenditure. I discuss the implications for other co-payment structures in section 5.8.

As a regulatory tool, information can go some way to helping deal with orphan conditions, where the patient population is too small to recover costs of research and trials in the standard monopoly period. Currently, regulators have special rules for drugs that can be found to treat orphan indications which generally rely on longer monopoly protection (Rosenthal, 2018). Prolonged monopolies with uniform pricing may lead to greater inefficiency while prolonging it under Indication-based pricing may result in longer periods of destruction of consumer surplus. As I show, the ability to incentivise testing these drugs through reinvestment of surplus may provide another solution. A possible combination of prolonged monopolies with the mechanism presented in this paper may be an optimal solution in some cases.

This paper presents a mechanism to deal with questions about how much information about individual patients should be made available to manufacturers. This can be expected to become a matter of growing significance in the future due to developments such as the advent of precision medicine, which allows for identification of drug effectiveness at a more individualised level, and improvements in drug re-purposing (Pushpakom et al., 2019) which implies a growth in multi-indication drugs. Both can lead to more observable heterogeneity in consumer value for a single product. Thus, regulation of information and its use as incentives for manufacturers should be an area of focus.

From a short-run perspective, the same setup can also be interpreted as a robustness check of Indication-Based Pricing where payers could lie about patient indications, but are still concerned about future interactions with the manufacturer. They could lie about individual patient uses after indications have been revealed, but too much lying could result in less surplus in the future as manufacturers adapt by reverting to a uniform pricing system and the associated problems return. I discuss this in more detail in section 6.1.

## 2 Example

Let's look at an example that illustrates how market segmentation results in different outcomes in the short-run. I do this here to provide some background to how market segmentation impacts outcomes in a monopoly.

Assume a manufacturer knows that its drug can potentially be useful for three indications; label them indication 1, 2 and 3. It costs the manufacturer \$20 to conduct the trials for each indication and consequently publicly include it in the uses of the drug. As a simplification, assume the manufacturer is already certain of the outcome of trials from evidence generated externally but needs to do them to get approval.

From the consumer side, the willingness to pay for treatment of patients is determined by their indication and is \$1,\$2 and \$3, for indications 1,2 and 3 respectively.

Furthermore, the prevalence of each indication in the population is well known, such that everyone knows there are 35 patients with indication 1, 56 with indication 2 and 9 with indication 3.

| WTP | Number of Patients | Fixed Cost of Inclusion |
|---|---|---|
| 1 | 35 | 20 |
| 2 | 56 | 20 |
| 3 | 9 | 20 |

Table 1: Indication Prevalence in Population

If the manufacturer has no information about individual consumers in this market, then the manufacturer's profit maximising action is to only test for indication 2 and charge a price of \$2, making a profit of (2 $\times$ 56 - 20 =) \$92.

Had the manufacturer included indication 1, the profit maximising price would still be \$2. So, the manufacturer would have incurred the cost of testing for indication 1 but not gained anything out of it as indication 1 consumers would be priced out anyway.

Indication 3 evokes a high willingness to pay, but the number of consumers with it is too low to recover the \$20 trial cost by charging \$2. So it's not profitable for the manufacturer to run trials for it.

Also note that it is socially efficient to test all three indications as the value of each indication is higher than the cost of testing (e.g. $3 \times 9 > 20$). In this particular example, consumer surplus is zero, however, it is generally possible to have inefficient outcomes with positive consumer surplus.

The efficiency problem could be solved by Indication-based pricing (first degree price discrimination), so the manufacturer is able to charge for each individual based on their indication. They would test for all indications and charge different prices to different patient groups. From a payer's perspective, the problem associated with such cases is that it always results in zero consumer surplus as all gains go to the manufacturer.

Suppose a regulator applies the following segmentation such that the manufacturer can only observe a patient's segment:

| Segment | Indication 1 | Indication 2 | Indication 3 | Total | Profit |
|---|---|---|---|---|---|
| 1 | 35 | 16 | 3 | 54 | 54 |
| 2 | 0 | 40 | 0 | 40 | 80 |
| 3 | 0 | 0 | 6 | 6 | 18 |
| Total | 35 | 56 | 9 | 100 | 152 - 60 = 92 |

Table 2: 3-Indication Example - Segmentation

The manufacturer now knows the distribution of different indications within a segment, but cannot observe the indications of individuals. Upon seeing a patient from segment 1, it knows they could have any of three indications. It notes that its maximum revenue from this segment is achieved by setting the price to \$1. It's best choices are then to charge \$2 in segment 2 and

\$3 in segment 3. The resulting profit is \$92 as before, while the consumer surplus has increased (maximised) to \$22 and output is maximised.

This segmentation is not unique as there are more ways of splitting up the market that achieve the same profit and consumer surplus. I refer to this as the basic case and elabourate on how to achieve other segmentations that account for distribution of consumer surplus among patients in section 5.7.

To illustrate how this mechanism impacts surplus in the long run, or even total output, let us look at an additional indication along with those stated above.

Indication 4 as shown in Table 2.

| Indication | WTP | Share of Patients | Fixed Cost of Inclusion |
|---|---|---|---|
| 4 | \$4 | 4 | \$20 |

Table 3: Inefficient Indication

Total Surplus from complete inclusion of this indication to those in example drops from \$114 to \$110. This is because gain from inclusion of this indication is \$16, which is less than the cost \$20.

From our previous segmentation, we know that other indication consumers have \$22 in surplus. If a regulator is concerned with increasing access to the medication then they could consider redistribution of this surplus towards paying the cost of testing for indication 4. The following segmentation achieves this so that consumer surplus is traded away to cover cost of trials.

Table 4: 4-Indication Example - Segmentation

| Segment | Ind 1 | Ind 2 | Ind 3 | Ind 4 | Total | Profit |
|---|---|---|---|---|---|---|
| 1 | 35 | 12 | 3 | 0 | 50 | 50 |
| 2 | 0 | 44 | 0 | 0 | 44 | 88 |
| 3 | 0 | 0 | 6 | 0 | 6 | 18 |
| 4 | 0 | 0 | 0 | 4 | 4 | 16 |
| Total | 35 | 56 | 9 | 4 | 104 | 172 - 80=92 |

If the inclusion of indication 4 adds more than \$4 to consumer surplus in the long run i.e. after the patent period ends then the regulator should do this segmentation. This would be achievable through Indication-based pricing.

Alternatively, even if the long run consumer surplus generated is less than the cost, a regulator may still wish to incentivise it to maximise health across the population. I discuss this more in section 5.6.

## 3 Literature Review

This paper is primarily an application of the principles of Information Design to a problems linked to heterogenity in effectiveness and willingness to pay in the pharmaceuticals market. As such it straddles both literatures: Pharmaceutical markets and Information Design.

A rudimentary method of individuals' information as incentives for manufacturers is Indication-based pricing. Several papers explore the impact of such a setup on overall welfare. Chandra and Garthwaite (2017), Goodman (2023) and Jiang et al. (2024) all present findings on possible negative impacts of such an approach on consumer well-being.

Goodman (2023) presents a theoretical framework including the patients, an insurer and a manufacturer to assess the impact of indication based pricing on welfare. She solves the manufacturer's problem for two indications with continuous demand curves for each indication to discuss the implications of the introduction of indication based pricing in that framework. I simplify the set-up by assuming that consumer valuation demand is determined solely but their indication and use this to provide more detailed answers for the case of any $n$ indications. Jiang et al. (2024) follows on from Goodman (2023) and shows the potential for loss in patient welfare in a framework with a single payer. I suggest a method to limiting the issues that arise from Indication-based pricing in both contexts and show how consumer surplus maximisation can be aligned with increasing efficiency in both the short and long-run.

Hlávka et al. (2021) and Lakdawalla and Sood (2013) are both concerned with dealing with potential inefficiencies caused by monopoly pricing in the pharmaceuticals market. Both address issues that arise in the patent period. I show that the results need not align with efficiency in the long run.

Within the patent period, Lakdawalla and Sood (2013) show how health insurance acts as protection against inefficiencies in the market by acting as a two-part pricing contracts. They address the setting of insurance in a free market like the US and essentially relies on giving away consumer surplus to manufacturers as incentives, in that sense it is a similar outcome to Indication-based pricing. Empirically, they are able to show that the impact of monopoly pricing on efficiency is much lower for drugs covered by insurance than without. I show how to use information about patients to achieve this with or without insurance and while maximising consumer surplus both in the long and short-run. I also show how a higher maximum achievable output is possible using market forces and the appropriate information structure.

Staying in the short-run, Hlávka et al. (2021) argue that efficient outcomes can be achieved through uniform pricing if the price is set as a result of Nash Bargaining between a single payer and the manufacturer. Lakdawalla and Sood (2013) makes a similar case of Nash Bargaining between insurers with monopsony power and single monopolists in their paper. This is similar to the concept of Weighted Average Pricing, which is also under discussion as an alternative method of pricing multi-indication drugs. If the incidence of disease in the population is well known then both systems are equivalent in terms of profit and consumer surplus. My paper can provide a mechanism that the Nash Bargaining is abstracting away from in the short-run. As such, it provides bounds on surplus division between both and can help a lower single price that payers should agree on.

Pushpakom et al. (2019) provides a good overview of the current state of drug re-purposing. They claim that it has gone from being 'largely opportunistic and serendipitous' to become much more systematic. They also highlight the fact that since it is cheaper and quicker to market, manufacturers with active patents on a drug can actively look to pursue this as an avenue for more revenue. However, while re-purposing costs a fraction of new drug development and is quicker, its costs are still not negligible. The choice of potential indications to pursue is therefore a strategic one and must be aligned with value for manufacturers. These facts feed into the assumptions of my mechanism. They also point to issues of incentives for the manufacturer.van der Pol et al. (2023) specifically highlight the issue of a lack of incentives for research into drug repurposing when drugs have gone out of patent. This establishes the need for optimising funding research while drugs are still in patent.

From the perspective of Information Design, we can also think about information provided to the manufacturer in terms of informativeness of Blackwell experiments. As explained by Tamuz (2024), the expected value to the manufacturer of receiving a signal about a consumers indication is the potential change in revenue from receiving that signal. A random signal is, of course, of no value. The upper limit to the value of signals is then achieved with noiseless information about each consumer, which allows the manufacturer to do Indication-Based pricing. Since willingness to pay for a drug is closely tied to patient indication, more information about the indication increases the manufacturer's value for information. The mechanism in this paper balances the manufacturer's value for consumer information with the cost of revealing potential uses, freeing up the remaining gain for reinvestment into research to maximise total surplus or simply through distributing it among consumers during the patent period. In this context, I show that Indication-based pricing results in "overpaying" the manufacturer by providing too much information and harming payers.

From the Information Design literature, I primarily use tools outlined by Bergemann and Morris (2019) which provides a comprehensive overview of the field. They identify two interpretations of the information design as a field; one with an actual designer or a metaphorical designer. This paper applies the principle of a regulator acting as a literal designer where it is reasonable to assume it has access to information about patients that manufacturers do not have or are restricted from using. For countries with a centralised state health system, this would most likely be the state insurer/payer responsible for dealing with pharmaceutical companies. For countries without a state insurer, it could potentially be a government regulatory body or the insurance companies themselves. As discussed in more detail later, the results can also be used to provide insight into the robustness of Indication-based pricing if we apply the concept of a metaphorical designer.

The mechanism extends and applies Bergemann et al. (2015) which takes a metaphorical design approach to highlight the existence of multiple potential consumer and producer welfare outcomes based on market segmentation in a monopoly. It also provides the methodology for segmenting a market such that consumer surplus is maximised in a monopoly. I use this as a basis to apply to an actual designer, but with the addition of two important assumptions. Firstly, I assume that the monopoly gives way to perfect competition in the future. Secondly, the designer is unaware of the entire potential pool of consumers that could be serviced and must design a mechanism without this knowledge while the monopolist can incur a fixed cost to inform the designer of the potential market. In doing so, I add the aspect of dynamic efficiency to their static framework. I also operationalize their results in a way that allows the system to be used as an incentive mechanism for the monopolist. As such, it has different implications for output, where, due to the decoupling of cost and output, it is possible to induce outcomes where output is higher than what is dynamically optimal for surplus maximisation.

This paper is also similar to Garcia and Tsur (2021) who apply Information Design to a problem of maximising welfare through a regulator in the insurance market. Their setting is a competitive market rather than a monopoly and also does not have fixed costs or a secondary post-monopoly period.

# 4 Model

There exist three sets of agents; a manufacturer, a regulator and a unit mass of patients (consumers). The situation is such that the manufacturer has already developed a drug that could be used for multiple indications but has not been tested for any of them.

New drugs that enter the market operate under patent protection for a fixed time period before they are classified as generics. This means that manufacturers operate as monopolists for a time period before their patent protection runs out and other competitors are allowed to enter the market - we can refer to these as the short-run and the long-run, respectively. I model this by assuming the existence of a first time period where the manufacturer operates as a monopolist followed by an infinite number of periods under perfect competition where prices drop to marginal cost (zero). For the purposes of this model, all strategic moves are carried out in the first period, while the remaining periods serves only to accrue discounted payoffs based on action in the first period.

Assume the existence of a total of $n$ indications $i \in D = \{1, .., n\}$ the drug can be used to treat, where $D$ is finite and countable. $D$ is the manufacturer's private information. Thus, in the first period, the manufacturer can run trials to reveal indication $i$ to the regulator and consumers at a cost $c_i$. This is a simplification of the testing procedure where trials serve as revelation costs. Let $c = (c_1, c_2, ..., c_n)$ denote the vector of costs. Assume further that there is a steady population of consumers for each period. Hence, there are $x_i$ consumers in indication $i$ such that $\sum_{i=1}^{n} x_i = 1$ in all periods. Let $x = (x_1, ..., x_n)$.

A consumer's valuation of the drug is based solely on the indication the individual consumer will be using it for. We normalise the value of good health to all patients as zero. Consumers in indication $i$ have a utility of $0 - v_i$ without using the drug. Treatment with the drug brings this up to zero, but incurs a price $p$. Hence the utility function for consumer $i$ is given by

$$u_c = \begin{cases} -p & consumes \\ -v_i & doesnt \end{cases}$$

This implies that patients with indication $i$ are willing to purchase the drug as long as $p \leq v_i$. This results in effective utility being $u_c = -\min\{v_i, p\}$. Furthermore, the consumer surplus for each indication $i$ consumer is then $v_i - p$. Valuation of the drug varies across the indications but is stable within them. Let $V = \{v_1, ..., v_n\}$ be the set of all valuations for indications in $D$ such that $v_{i+1} > v_i$. Consumers in an indication do not buy the drug unless the manufacturer runs trials for their specific indication.

In the first period, the manufacturer looks to maximise profit, defined as the total revenue it gains minus the cost of testing for each indication. If the manufacturer is able to charge a different price for each indication then it would simply test all the indications in which the total gain from sales exceeded the cost from testing. Profit would then be $\sum_{\{i|v_i x_i \geq c_i\}}(v_i x_i - c_i)$. This drives consumer surplus to zero in the first period. In subsequent periods, perfect competition ensures that the manufacturer's profit is zero.

If the manufacturer is forced to charge a single price, then the increase in revenue from each additional indication may not be enough to offset the cost of its testing. This may prevent the manufacturer from testing all indications whose social benefit exceed their cost. The manufacturer's exact actions in this situation are discussed and presented as a result in section 5.4. From the consumer's perspective, a benefit of uniform pricing over Indication-based pricing is that for any indications that are tested, the manufacturer may not be able to capture the entire surplus in the first period.

The regulator does not know $D$ beforehand and only finds out about the indications revealed by the manufacturer after testing, where $M \subseteq D$ is the set of indications revealed by the manufacturer. The regulator does know about the number and valuation of consumers in each indication such that once $M$ is revealed it automatically knows the valuations, $v^M = \{v_i|i \in M\}$, and share of patients, $x^M = \{x_i|i \in M\}$, who could use the drug in both periods. It does not know that $\sum_{i \in D} x_i = 1$ so it cannot know if the manufacturer is hiding indications.[2] Furthermore, $c_i$ is public knowledge, such that the regulator and the manufacturer know how much it costs to run trials for a particular indication, so once $M$ is revealed the regulator also finds out $c^M = \{c_i|i \in M\}$. [3]

The regulator's concern is patient welfare, such that it wants to maximise consumer surplus; in cases where trade leads to zero consumer surplus it prefers trade over no trade. Additionally, regulators have other distributional considerations which I discuss briefly in section 5.7.

In order to achieve its goals, the regulator can carry out market segmentation in the first period such that individual usage of the drug are grouped together i.e. each segment can comprise a mix of patients with different indications, where the manufacturer knows the composition of the group but not the individual characteristics of its members, forcing it to charge a single price in each segment. Knowing how this segmentation is going to occur, the manufacturer can choose which indications to test for.

After the monopoly period ends, all consumers get access to the drug at price equal to the marginal cost i.e. zero.

**Segmentation Algorithm:** I now define what a segmentation algorithm is. A segmentation algorithm maps a set of indications $M \subseteq D$, its corresponding valuations $v^M = \{v_i|i \in M\}$, share of patients $x^M = \{x_i|i \in M\}$, and the costs $c^M = \{c_i|i \in M\}$, to a set of segments, $S$.

I refer to individual segments as $s_j$, with each one identified by its profit maximising uniform monopoly price such that $s_j$ has the profit maximising price $v_j$. The segmentation is done such

[2] A way to think about it is to assume there is a universal set of all indications in the population that is a superset of $D$. Everyone has access to the valuations and population of patients in these indications, but lack information about $D$. Given any set of indications $K$, the regulator knows $v^K = \{v_i|i \in M\}$ and $x^K = \{x_i|i \in M\}$.

[3] The cost of running a trial is not as difficult to establish as other costs in the drug development process. Furthermore, while we do not explicitly use this fact, it is interesting to consider the relation between $v_i$,$x_i$ and $c_i$. Trials for small populations are going to be more difficult and consequently expensive as it is difficult to get participants. Similarly, drugs with big impacts (and consequently higher $v_i$) are going to be cheaper as they require a smaller test sample. Hence, on the whole, we can expect $\frac{\partial c_i}{\partial x_i} < 0$ and $\frac{\partial c_i}{\partial v_i} < 0$.

that no two segments have the same profit maximising uniform monopoly price. [4] (Since valuations are discrete, it is safe to assume that it is always optimal to choose a price from $V = \{v_1, ..., v_n\}$ for any given group). A segmentation is then defined by $t^M_{ji}$'s, the proportion of patients in $i \in M$ assigned to segment $s_j$, such that $s_j = \{t_{ji}x_i\}^n_{i=1}$.

A segmentation algorithm is then a mapping $T(M, v^M, x^M, c^M)$ to vector $t^M = (t^M_{ji})$'s where $i \in M$ and $j \in M$. [5] Segment $s_j$ exists in $S$ iff $\sum_{i \in M} t^M_{ji} \neq 0$.

**Timing:** Having established what a segmentation algorithm is, I formally state the timing of the game as follows:

**Period 1:**

1 Manufacturer observes market structure, $x$, valuations, $V$ and costs $c$.

2 Regulator commits to algorithm to formulate segmentation, $T()$.

3 Manufacturer decides what to test and consequently declares list of indications $M \subseteq D$.

4 Regulator observes $M$ and consequently knows $v^M, x^M$ and $c^M$. Using this, patients assigned segments based on predetermined segmentation algorithm.

5 Manufacturer charges price for each group based on its constituents and payoffs are realised. (Patients in each segment buy the drug only if the price is below their valuation).

**Period 2 onwards:**

1 Price drops to zero for all consumers. Consumers with indications $i \in M$ get a payoff $v_i$, while the manufacturer gets a payoff of zero.

I assume that in case of indifference, the manufacturer always opts to include more indications than less. Even without this assumption, the manufacture can be nudged in this direction by promising an infinitesimal increase in profit, $\varepsilon$, for the inclusion of each indication in $M$.

The regulator's commitment can be seen as credible either through legislation or simply considered as part of a repeated game between the regulator and the manufacturer. I discuss this in greater detail in section 6.1.

**Manufacturer's Problem:** Given that the regulator will have committed to an algorithm $T(M, v^M, x^M, c^M)$, the manufacturer knows the segmentation for each potential $M$, $t^M$. Thus, the manufacturer's problem is simply to find $M$ that maximises profit.

$$\pi^*_M = \max_{\{M|M \subseteq D\}} \sum_{i \in M} \sum_{\{j|s_j \in S\}} v_j t^M_{ji} x_i.1_{\{v_j \leq v_i\}} - \sum_{i \in M} c_i$$

**Regulator's Problem:** Thus the regulator must develop an algorithm to ensure that the segmentation results in the maximisation of a social welfare function $U()$. For now, I state it simply as total consumer surplus. I later move towards other formulations of $U()$ that are concerned with distributional considerations among patients during monopoly pricing.

[4]This is not a requirement but a natural outcome of the algorithm.

[5]Thus, in example in Table 2 we had $t^D = (t^D_{11} = 1, t^D_{12} = 0.39, ..., t^D_{33} = 1)$ and so on.

The maximisation is then defined by

$$u^* = \max_{\{t_{ji}^M | i,j \in M\}} \sum_{i \in M} \sum_{j \in M} t_{ji} x_i (v_i - v_j) + \frac{\delta}{1-\delta} \sum_{i \in M} v_i x_i$$

where $\delta$ is the per period discount factor for all agents.

There are four groups of constraints are as follows:

**Marginal Constraint** ensure that all patients are assigned to some segment and that no double assignment occurs.

$$\sum_{j \in M} t_{ji}^M = 1 \quad \forall i \in M$$

**Non-negative Utility** simply states that since no purchases will be made where the price is set higher than the value to an individual. Negative contributions cannot be made to the total consumer surplus.

$$t_{ji}^M x_i (v_i - v_j) \geq 0 \quad \forall i, j \in M$$

**Obedience Constraints** the regulator must ensure that it is in the manufacturer's interest to charge the price suggested by the segmentation. Thus, the segmentation must satisfy the following the obedience constraints, that suggest that the manufacturer cannot increase profits in each group $j$ by charging a price different to $v_j$.

$$\sum_{i \in M} v_j t_{ji}^M x_i 1_{\{v_j \leq v_i\}} \geq \sum_{i \in M} v_j'.t_{ji}^M x_i 1_{\{v_j' \leq v_i\}} \; \forall v_j, v_j'.$$

**Definition 4.1.** Define $M^*$ as set of indications whose revelation maximises consumer surplus i.e. the regulator would try to induce $M = M^*$.

**Inclusion Constraint** ensures that it is in the manufacturer's interest to reveal the optimal subset of indications. The regulator can achieve this by making the manufacturer indifferent between revealing his optimal choice of indications and $M^*$. Effectively, this can be achieved by giving a profit guarantee to the manufacturer

$$\pi_{M^*}^* \geq \max\{\pi_K^* | K \subseteq D\}.$$

This constraint actually relies on the fact that the regulator should be made better off by excluding indications the regulator wants or including any indications that they don't. [6]

**Definition 4.2.** Define $I, I^f, I^c \subseteq D$ such that $I = \{i | i \in D, v_i x_i \geq c_i\}$, $I^f = \{i | i \in D, v_i x_i < c_i \leq v_i x_i \frac{1}{1-\delta}\}$ and $I^c = \{i | i \in D, v_i x_i \frac{1}{1-\delta} < c_i\}$.

$I$ is then the subset of indications for whom the cost of testing and inclusion is lower than the total gain $v_i x_i$ in the first period, $I^f$ is the subset of indications for whom the cost of testing

[6] I coin the phrase Inclusion Constraint that could be perceived as an Incentive Compatibility constraint or a Participation Constraint or even something else based on how it is framed. Assume $M^* \subseteq D$ is the $M$ that maximises the regulators objective function. Then if we consider $M^*$ to be the manufacturer's type then this can be considered an Incentive Compatibility constraint. On the other hand, as the purpose of the constraint is to guarantee a minimum welfare to the Manufacturer so as to ensure their participation in the mechanism, it could also be seen as a Participation Constraint.

and inclusion is higher than the gain in first period, lower than the total gain $v_i x_i$ over all periods and $I^c$ are the indications that have a cost higher than the total surplus they generate over all periods.

As we will see, the regulator's primary aim is consumer surplus maximisation then $M^* \supseteq I^f$ is the optimal outcome.

# 5 Results

I start by discussing the inclusion constraint and the profit that a manufacturer must be guaranteed by revealing $M^*$ before moving onto defining the characteristics of $M^*$.

## 5.1 Minimum Achievable Profit

I start by defining two terms that help bring clarity to the problem. Since the manufacturer's only earns profits during patent protection, for now we maintain focus on only the first period.

**Definition 5.1.** Define $\pi^U_K$ as the highest profit that a manufacturer can achieve by charging a uniform price over a set of indications, $K \subseteq D$.

Thus, if the segmentation algorithm puts all consumers in the same segment, the manufacturer is guaranteed $\pi^U_M$.

**Definition 5.2.** Define $P \in D$ such that $\pi^U_P \geq \pi^U_K$ for any $K \subseteq D$.

$P$ is thus the set of indications that generates the maximum profit if the algorithm put all consumers in the same segment; this profit is denoted by $\pi^U_P$.

Note that the manufacturer makes decisions in two dimensions; setting the price and choosing $M$. This fact can help us establish a lower bound on the profit that the manufacturer must receive as a result of the segmentation exercise.

For any given $M$, the manufacturer always has the option to set the same price in all segments. This is the equivalent of treating all segments as one, with the $\pi^U_M$ being the resultant profit. Since any segmentation provides weakly greater information about individuals consumers, it should always result in weakly greater profits for the manufacturer. Another way of saying this is that segmentation of $M$ cannot reduce profit below $\pi^U_M$. Thus, we establish $\pi^U_M$ as the lower bound for any set $M$.

The second dimension is the choice of $M$ that maximises $\pi^U_M$. Since we know that $M = P$ maximises $\pi^U_M$, we know that the lower bound on profit for the manufacturer is $\pi^U_P$, the highest possible profit achievable from uniform pricing. So, regardless of the segmentation algorithm, the manufacturer can always ensure $\pi^U_P$ for itself.

**Proposition 1.** For any $M$ it is always possible to get a segmentation that:

1 achieves a total surplus of $\sum_{i \in M}(v_i x_i - c_i)$ , and

2 achieves any profit on the interval $[\pi^U_M, \sum_{i \in M}(v_i x_i - c_i)]$

*Proof.* This result is established by Bergemann et al. (2015) without fixed costs. Adding fixed costs makes no difference to the result. □

So we establish that there exist values of $t_{ji}^M$'s that satisfy the regulator's constraints and the following two conditions

$$\pi_M^U \leq \sum_{i\in M}\sum_{j\in M} t_{ji}^M v_j x_i - \sum_{i\in M} c_i \leq \sum_{i\in M}(v_i x_i - c_i)$$

and

$$\sum_{i\in M}\sum_{j\in M} t_{ji}^M v_j x_i 1_{\{v_j\leq v_i\}} - \sum_{i\in M} c_i + \sum_{i\in M}\sum_{j\in M} t_{ji}^M (v_i - v_j) x_i 1_{\{v_j\leq v_i\}} = \sum_{i\in M}(v_i x_i - c_i).$$

We are now ready to state out first theorem which outlines the restrictions on achievable $M^*$.

**Theorem 1.** A segmentation always exists that ensures the optimal set of indications (as required by the regulator), $M^*$ and efficiency within the set of indications iff $\sum_{i\in M^*}(v_i x_i - c_i) \geq \pi_P^U$. This is always true if $M^* = I$.

*Proof.* In order for the manufacturer to report $M^*$, it must be that

$$\pi_P^U \leq \sum_{i\in M^*}\sum_{j\in M^*} t_{ji}^{M^*} v_i x_i - \sum_{i\in M^*} c_i$$

The lower bound on the first condition stated above becomes irrelevant as, by definition, $\pi_{M^*}^U \leq \pi_P^U$. Thus any choice set $M^*$ can only be feasible if $\pi_P^U \leq \sum_{i\in M^*}(v_i x_i - c_i)$.

For $M^* = I$, the upper bound $\sum_{i\in I}(v_i x_i - c_i)$ is highest achievable surplus from any subset in $D$ because the value of each indication is higher than the cost. Consequently, no uniform monopoly profit could exceed the total surplus. Hence, $\pi_P^U \leq \sum_{i\in I}(v_i x_i - c_i)$ is always satisfied. $\square$

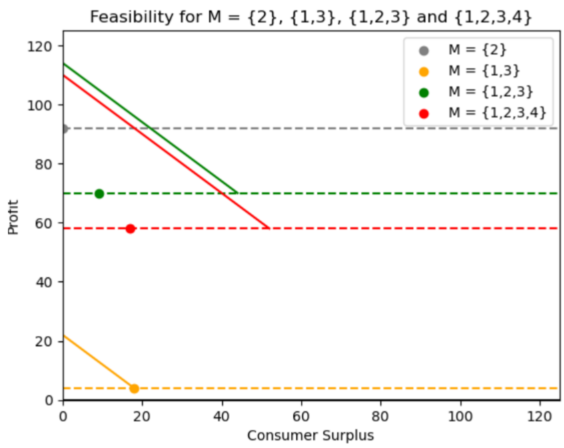



Figure 1: Potential Outcomes based on $M = \{2\}, \{1,3\}, \{1,2,3\}, \{1,2,3,4\}$

Figure 1 illustrates the reasoning behind Theorem 1 (and upcoming ones) using the example from Section 2. In this case, $D = \{1,2,3,4\}$, $V = \{\$1, \$2, \$3, \$4\}$, $x = (35, 56, 9, 4)$ and $c_i = \$20$ for all indications. It is obvious that $I = \{1,2,3\}$ and $I^c = \{4\}$.

Each colour denotes a different set $M$, the dashed line shows the value $\pi^U_M$ for each set. Since we have already seen the profit maximising choice of the manufacturer earlier, we know that $P = \{2\}$, hence the grey line denotes $\pi^U_P$. The solid lines show the efficient division of total surplus achievable in each $M$ through some segmentation; the y-intercept shows the maximum profit achievable through first-degree price discrimination, $\sum_{i\in M}(v_i x_i - c_i)$, while the lowest profit possible through segmentation in $M$ is given by $\pi^U_M$.

We note that there is no way for the regulator to induce the manufacturer to set $M = \{1, 3\}$, with or without segmentation. This is because the profit from an unsegmented $M = \{2\}$ is always higher. In fact, choosing any of the other sets in the diagram would always make the manufacturer better off than choosing $M = \{1, 3\}$ as the lowest achievable profit in those sets is higher than the highest achievable profit in $\{1, 3\}$. Thus we know that any feasible $M^*$ has at least some part of the solid line above $\pi^U_{\{2\}}$.

The dots mark the consumer surplus and profit that would be achieved in each $M$ without any segmentation. Here we see that $M = \{1, 3\}$ would achieve the highest consumer surplus without segmentation. However, the highest achievable consumer surplus after segmentation from a set $M$ occurs at the intersection of the solid and dashed lines. For $M = \{2\}$, we know that this is zero (as it would be for any singleton).

## 5.2 Optimal Outcomes

**Definition 5.3.** Let $w^* = u^* + \pi^*_M$ be the total surplus achieved as a result of the algorithm.

For simplicity I refer to the segmentation that maximises consumer surplus as the *'algorithm output'* from here on. Furthermore, I refer to Indication-Based Pricing that emerges as a result of the algorithm output as '*ex-post* Indication-Based pricing' while 'Indication-Based pricing' by itself refers to a commitment to price consumers solely based on their indications.

We now turn our attention to choice of $M^*$. In order to establish optimal outcomes, in terms of the regulators objectives and overall efficiency we must now consider all periods. We start by establishing some baselines.

**Lemma 1.** Indication-based pricing is efficient if and only if $I^f = \varnothing$.

*Proof.* For any indication under Indication-based pricing, the monopolists return is $v_i x_i - c_i$. Hence it only tests indications in $I$. However efficiency requires all indications in $I \cup I^f$ be traded, as $c_i \leq \frac{v_i x_i}{1-\delta}$ for $i \in I \cup I^f$. So unless $I^f$ is empty, the outcome from Indication-based pricing is inefficient. □

This result shows the issue of manufacturers having a shorter time horizon than the regulator. While Indication-Based pricing leads to efficiency over the patent period, it does not resolve inefficiency in the long run. Thus, indication-based pricing can only achieve efficiency if no indications exist whose benefit accrues over the long run, either due to low $\delta$ which puts any indications outside $I$ into $I^c$, or $D = I$, such that $I^f$ and $I^c$ are both empty.

We now return to our example from section 2 with four indications, $D = \{1, 2, 3, 4\}$ with two cases. Initially, assume that $\delta < 0.2$ so that $I = \{1, 2, 3\}$, $I^f = \varnothing$ and $I^c = \{4\}$. In this case, it is possible and efficient to do segmentation on $M = I$ that results in $u^* = \$22 + \$174\frac{\delta}{1-\delta}$

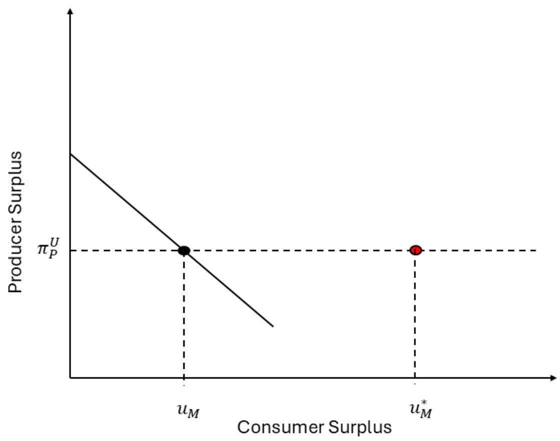


Figure 2: Short-term vs. Long-term Consumer Surplus

as shown in section 2 while meeting the inclusion constraint. This is the highest achievable consumer surplus in this setup and maintains efficiency, establishing $M^* = I$.

Now assume that $\delta > 0.2$ so that $I = \{1, 2, 3\}$, $I^f = \{4\}$ and $I^c = \varnothing$. It is possible to induce the manufacturer to include 4 in $M$ if the regulator conditions the segmentation in Table 4 on its inclusion. The resulting total surplus is now higher than it would be with indication-based pricing, where $\pi^* = \$92$, $u^* = \$18 + \$188\frac{\delta}{1-\delta}$ and $w = \$100 + \$188\frac{\delta}{1-\delta}$. In fact, both total surplus and consumer surplus have been maximised in this situation.

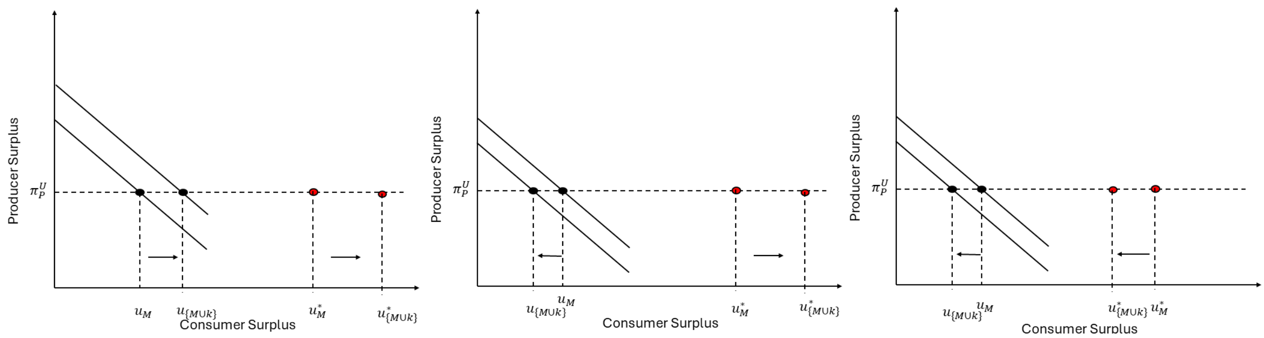
Note that while total surplus from segmentation would have coincided with that from indication-based pricing when $\delta < 0.2$ – with higher consumer surplus through segmentation – for the case where $\delta > 0.2$ the total surplus through segmentation is higher. Note further that the consumer surplus derived from the monopoly period is lower in this case. Essentially, through conditional segmentation we are able to induce the manufacturer to reveal indication 4 by making up the shortfall $(20 - 4 \times 4) = 4$ through redirecting some of the consumer surplus generated in the first period towards the cost of inclusion of indication 4. This allows consumers to benefit from this indication in all subsequent periods. Thus, conditional segmentation has allowed for the transfer of consumer surplus from the first period to the others.

I now generalise and explain these outcome and formally present the results. Figure 2 highlights the achievable consumer surplus by splitting it up into short-term consumer surplus attained in the monopoly period, $u_M$ and the overall long-term period, $u_M^*$. Thus for any $M$, $u_M^* = u_M + \frac{\delta}{1-\delta}\sum_{i \in M} v_i x_i$.

**Lemma 2.** Inclusion of indication $i \in I^f$ leads to an increase in total consumer surplus.

*Proof.* We note that since $u_{\{M \cup k\}} = u_M + (v_k x_k - c_k)$ and $u^*_{\{M \cup k\}} = u_{\{M \cup k\}} + \frac{\delta}{1-\delta}\sum_{i \in \{M \cup k\}} v_i x_i$. The difference in total consumer surplus from including $k$ is given by $u^*_{\{M \cup k\}} - u^*_M = (v_k x_k - c_k) + \frac{\delta}{1-\delta} v_k x_k$, so we note including $k$ increases overall consumer surplus if $k \in I^f$ □

Figure 3 shows the impact of including indication $k$ based on its characteristics. (In Figure 3a, I assume that the indication included does not impact $P$, in others this must always be true - this has no implication for the result). Hence we also find that any indication from $I$ increases both short and long-term consumer surplus.




(a) $k \in I$ (b) $k \in I^f$ (c) $k \in I^c$

Figure 3: Impact of including $k$ on Short-term and Long-term Consumer Surplus

I now set about turning these observations into general results. Let's start with a focus on finding $M^*$.

**Definition 5.4.** Define $\underline{v}_K$ be the valuation of the lowest indication in a set of indications, $K$.

**Lemma 3.** The regulator should always induce all indications in $I$.

*Proof.* The proof is done in two parts. First we show that the gain in profit for the manufacturer is weakly less than the gain in total achievable surplus: Assume some $M \subset I$ with uniform profit $\pi_M^U$. If another indication $k$ is added to make this $M'$ then the increase in total surplus is $v_k x_k - c_k$. The change in uniform profit,

$$\pi_{M'}^U - \pi_M^U = \begin{cases} \underline{v}_M x_k - c_k & v_k > \underline{v}_M \\ v_k x_k - c_k - (\underline{v}_M - v_k) \sum_{j \in M} x_j & v_k < \underline{v}_M \end{cases}.$$

Clearly, this is less than $v_k x_k - c_k$. The second part appeals to Theorem 1 which shows that there exists some segmentation that allows the regulator to appropriate the part of additional surplus not captured by the manufacturer. □

**Corollary 1.** Any algorithm that seeks to maximises consumer surplus will have to induce $M^*$ such that $M^* \cap I^c = \varnothing$.

*Proof.* Consider an indication $k \in I^c$. The gain in profit from including this indication in $M$ must be $\pi_{\{M \cup k\}} - \pi_M \geq c_k$. The maximum potential gain in consumer surplus is $\frac{v_k x_k}{1-\delta} < c_k$. For any efficient outcome this implies a drop in consumer surplus, which means that the regulator will dissuade inclusion of $k$. □

**Corollary 2.** The regulator induces $M^* = I$ if and only if $I^f = \varnothing$.

*Proof.* Follows obviously from lemmas 1 and 3. □

We've established that the inclusion of indications from $I$ increase the total surplus in the first period, and that it is possible for the regulator to appropriate some part of the increase towards consumers surplus while maintaining efficiency. It can do so in two ways. If it only cares about the short-term, or if there are no indications that are efficient only in the long-term, then the regulator realises the maximum consumer surplus in the first period. This corresponds to the maximum total and consumer surplus achievable in this situation. The total surplus achieved is equal to that achieved by Indication-based pricing.

The other way the regulator can use the appropriated surplus in the first period is as incentives for investment into more trials. So, the manufacturer receives the surplus again as revenue if it bears the cost of certain trials. This maximises consumer surplus in the future if the trials include indications from $I^f$. Furthermore, addition of indications from $I^f$ or $I^c$ to $M$ leaves $\pi_P^U$ unchanged.

The manufacturer can easily be dissuaded from including any indications $k \in I^c$ by assuming the set $M \backslash k$ as input and putting all patients with indication $k$ in a separated segment $s_k$. The profit maximising move from the manufacturer will then be to charge them $v_k$, which will not result in them offsetting their costs. Knowing this, the manufacturer is dissuaded from including such indications in the first place. I present this formally when I describe the regulator's algorithm in section 5.5.

**Theorem 2.** If

1 $I^f \neq \varnothing$,

2 $M^* \subseteq \{I \cup I^f\}$, and

3 $\exists i \in I^f$ such that $\{I^f \cap M^*\} \neq \varnothing$ and $\sum_{i \in \{I^f \cap M^*\}}(c_i - v_i x_i) \leq \sum_{i \in I}(v_i x_i - c_i) - \pi_P^U$

then the algorithm output achieves a greater total surplus and consumer surplus than Indication-based pricing.

*Proof.* The proof is as follows.

(1 and 2) If $I^f = \varnothing$, then we have established from Lemma 1 and Corollary 2 that $M^* = I$ is induced by both Indication-based pricing and the regulator with segmentation. From Lemma 1 we establish that Indication-based pricing is efficient, while from Proposition 1 we establish that there exists an efficient segmentation with higher consumer surplus than Indication-based pricing. Lemma 2 shows that including indications from $I^f$ increases total surplus.

3 From Lemma 3, we know that indications from $I$ serve to maximise surplus in first period. $\pi_P^U$ of this surplus goes to satisfying the inclusion constraint. The remaining can be used to cover costs of indications from $I^f$ that cannot be covered by revenue generated in the first period.

□

**Corollary 3.** If $\sum_{i \in I^f}(c_i - v_i x_i) \leq \sum_{i \in I}(v_i x_i - c_i) - \pi_P^U$ then the algorithm output maximises total surplus and consumer surplus

*Proof.* Building on the Theorem 2, we find that if all indications in $I^f$ can be included in $M$ feasibly then total surplus has been maximised, as has consumer surplus. All surplus from $I$ and $I^f$ has been realised and indication from $I^c$ would reduce total surplus. □

In certain cases, the regulator might want to include indications from $I^c$ because of a priority for access over consumer surplus. I address this in section 5.6.

### 5.3 Optimal Profit

We have established that $\pi_P^U$ plays a critical role in this mechanism. Thus, it is important to focus on what the manufacturer's profit maximising set of indications would be and the profit it would generate. Simply put, the segmentation algorithm relies on assuring the maximum profit achievable in a uniform price setting within any subset $M$. Thus, it establishes the optimal desired set $M^*$ based on the regulator's priority and then chooses a segmentation that guarantees $\pi_P^U$ within that $M$ while maximising surplus. I go into detail for the characteristics of $\pi_P^U$ and provide an algorithm to find $P$ in section 5.4. For now I present one characteristic of $P$ that is useful for the next theorem.

**Definition 5.5.** Let $\pi_K = \underline{v}_K \sum_{i \in K} x_i - \sum_{i \in K} c_i$ for any subset $K \subseteq D$.

**Lemma 4.** For any $K \subset D$, there exists $K' \subseteq K$ that achieves a weakly higher uniform monopoly profit with $\underline{v}_{K'}$ being the optimal uniform monopoly price.

*Proof.* If the uniform monopoly profit without segmentation for $K$, $\pi_K^U$ was achieved by setting the price at $v_k > \underline{v}_K$ then for any positive cost $c_l$ for $k > l \in K$, profit will always increase by moving to $K' \subset K$ where $K' = \{j : j \in K, j \geq k\}$. □

Intuitively, this makes sense as, if the profit maximising uniform price is higher than the lowest value indication, then the cost of including that indication is being paid but no sale is being made for that indication. As a result, it would be optimal to not include that indication in the first place to avoid that cost. Thus, we can narrow our search for maximum achievable uniform monopoly profits to just the set of $\pi_K$ for $K \subseteq D$.

**Lemma 5.** The profit maximising price to achieve $\pi_P$, $\underline{v}_P$, must be the valuation of an indications from $I$.

*Proof.* Suppose some set $K$ with the lowest indication $k \in I^c$ such that $\underline{v}_K = v_k$ and second lowest indication $k'$. Then $\pi_K = v_k \sum_{i \in K} x_i - \sum_{i \in K} c_i$. Removing $k$ results in an increase of greater than $c_k - v_k x_k > 0$. □

So we can now narrow our focus down to $\pi_K$ for $K \subseteq I$.

Furthermore, what lemmas 1 and 3 show is that the algorithm chooses an efficient outcome within $M^*$. This implies that it must be that $u^* + \pi^*_{M^*} = \sum_{i \in M^*} v_i x_i - c_i$. Thus for any efficient outcome, there is a zero-sum game between achieving consumer surplus and the manufacturer so the inclusion constraint is always going to bind. It further implies that for any efficient outcome the achievable consumer surplus is also known if the producer surplus is known.

**Theorem 3.** A segmentation is feasible and maximises consumer surplus within $M^*$ if and only if:

$$\sum_{\{t_{ji} | j < i, i, j \in M^*\}} t_{ji} \leq 1 \quad \forall i \in M^*$$

$$\sum_{j < i < k} t_{ji} v_j x_i \geq \sum_{i \geq k} t_{ji} (v_k - v_j) x_i \quad \forall k, j \in M^* \ s.t. \ k > j$$

$$\sum_{\{i \geq j | i,j \in M^*\}} t_{ji} v_j x_i - c_j = \max\{\pi_K | K \subseteq I\}$$

These constraints refer to $\frac{m(m-1)}{2} - 1$ free variables, where $|M| = m$.

*Proof.* The non-negativity constraint implies that $t_{ji} = 0$ for $j > i$ hence we are left with $\frac{m(m+1)}{2}$ non-zero variables. From the marginal constraint we know that for each $i$, $t_{ji}$'s must sum to 1. This implies that there are $i-1$ free $t$'s for each $i$ while one variable simply picks up the residual. We can define $t_{ii} = 1 - \sum_{j \in M^*} t_{ji}$ for all $j < i$. Hence we combine the first non-negativity constraint and the marginal constraint to arrive at the first inequality expressed here. If this constraint binds for any $i$, we know that $s_i$ does not exist.

Furthermore, we not that obedience constraints that deviate to lower prices are never binding. For the case where $v'_j = v_k < v_j$, the obedience constraint becomes

$$\sum_{k \leq i < j} t_{ji}(-v_k) x_i 1_{\{v_j \leq v_i\}} + \sum_{i \geq j} t_{ji}(v_j - v_k) x_i 1_{\{v_j \leq v_i\}} \geq 0$$

From non-negative utility constraint, we know that $t_{ji} = 0$ for $j > i$. Thus, we are left with

$$\sum_{i \geq j} t_{ji}(v_j - v_k) x_i \geq 0$$

which is true by definition for any value of $t_{ji}$ so the constraint is not binding. Due to the obedience constraints, the manufacturer can expect to receive at least $\pi_M^U$ for any $M$.

The inclusion constraint means that they can expect to receive $\pi_M^* \geq \max\{\pi_M^U, \pi_K^U\}$ for any $K \subseteq D$. Hence, the inclusion constraint ensures that it is costless for the manufacturer to include all indications in $M^*$. From Lemma 4 we know that $\pi_M^* = \max\{\pi_M, \pi_K\}$ for $K \subseteq M$. This means that at $M = M^*$, the inclusion constraint becomes $\pi_{M^*}^* = \max\{\pi_M^*, \pi_K\}$. As $\pi_M^* \geq \pi_K \quad \forall K \in I$ this could be expressed as a series of $|P(M^*)| - 2$ constraints, (one of which will bind, unless $\pi_{M^*} \geq \pi_K \quad \forall K \in P(M^*)$).

However, if we separate the process of finding $\max\{\pi_K | K \subseteq I\}$ as we do in section 5.4 this becomes a single equality constraint. The last equality constraint removes one more degree of freedom, leaving $\frac{m(m-1)}{2} - 1$ free variables. □

Efficiency is ensured by the non-negativity constraint wherein, once $M^*$ is established, we ensure that all consumers are served, as a result of which there is no deadweight loss. Since there is always a trade, we can further simplify the problem and characterise the solution of consumer surplus maximisation. This allows us to move towards more considerations later on, where the regulator may now have other preferences of $U()$ which we explore in section 5.7.

What this further shows is that the inclusion constraint provides the (achievable) upper-limit on the consumer surplus. Hence, we get a plane in $\frac{m(m-1)}{2}$ dimensions where the coordinates are $t_{ji}$'s for $i > j$. The obedience constraints then put restrictions on choice of $t_{ji}$'s within this plane.

### 5.4 Profit Maximising Set of Indications

We now turn towards the exercise of finding $P$ such that $\pi_P = \max\{\pi_K | K \subseteq I\}$. As we have already established we look towards providing the manufacturer the maximum uniform monopoly profit they would achieve from selecting any subset of $D$. Hence, we are now looking to solve the problem of finding the set $P \subseteq D$ that maximises profit in a uniform monopoly price setting.

**Definition 5.6.** Define effective cost $\tilde{c}_i = \frac{c_i}{x_i}$ for all $i$.

While we operate in a setup with sunk costs, at the phase of deciding $P$ we use $\tilde{c}_i$ as a substitute for marginal cost in the region of indication $i$.

Assuming that no indications could be excluded, we can then see the total change in revenue from including an indication $i$ after $i+1$ as

$$v_i f(i) - (v_{i+1} - v_i)(1 - F(i))$$

where $f(i) = x_i$ and $F(i) = \sum_{j=1}^{i} f(i)$. This is the marginal revenue for the indication $i$. This is familiar and can be expressed as the Myersonian virtual valuation, $\hat{v}_i$ (Myerson, 1981) such that

$$\hat{v}_i f(i) = f(i) \left( v_i - (v_{i+1} - v_i) \frac{(1 - F(i))}{f(i)} \right).$$

Bulow and Roberts (1989) provide an interesting insight into the link between virtual valuations and marginal revenue, showing that if $1 - F(i)$ is interpreted a demand curve then the virtual valuation is the marginal revenue. Furthermore, as Myerson (1981) defines regular distributions as those for whom the virtual value function is monotonic, the same link is drawn by Bulow and Roberts (1989) for monotonic marginal revenue functions.

Thus, under assumptions of regular distributions, it is weakly beneficial for the manufacturer to include $i$ if $\hat{v}_i \geq \tilde{c}_i$. However, both Bulow and Roberts (1989) and Myerson (1981) refer to ironing as a way of dealing with irregular distributions. Now, since we have put no restrictions on $x$ and $c$, we must assume the existence of situations with irregular distributions and non-monotonic marginal costs. As such, we must look towards ironed values of both. The result below account for irregularity and non-monotonicity in such a similar way.

**Theorem 4.** For profit maximising set $P$, define an indexing scheme for elements within it such that $v^m$ refers to the valuation of indication with the $m$-th highest valuation in $P$ (and $c^m$ is the corresponding cost of inclusion). The necessary conditions for profit maximising set $P$ are:

1. $v^1 - (v^2 - v^1)\frac{x_2}{x_1} \geq \tilde{c}^1$ for $|P| \geq 2$
2. For any $m \leq |P|$, $v^1 - (v^m - v^1)\frac{\sum_{\{k:k \geq m, k \in P\}} x^k}{\sum_{\{k:k<m, k \geq 1, k \in P\}} x^k} \geq \frac{\sum_{\{k:k<m, k \geq 1, k \in P\}} c^k}{\sum_{\{k:k<m, k \geq 1, k \in P\}} x^k}$
3. $v^1$ is the profit maximising price.
4. For any $m \leq |P|$, $\tilde{c}^m \leq v^1$

*Proof.* 1 If $v^1 - (v^2 - v^1)\frac{x_2}{x_1} < \tilde{c}^1$ then profit is maximised by dropping $v^1$ from $P$.

2 relies on the Lemma 4, such that the lowest value in the set must be the profit maximising uniform price. Hence, for any valuation higher in set $P$, it must be profitable to move from currently charging $v^p$ for all indication in $P$ to charging $v^1$ and incurring the cost of including the new indications.

3 is established in Lemma 4.

4 reiterates the fact that for any indication included in $P$, the effective cost must be lower than the value of the lowest indication in $P$.

□

An implication of part 3 in Theorem 4 is that no indication from $I^c$ is ever in $P$. Hence, we know that $P \subseteq I$ and we focus solely on $I$ for the rest of the section.

Part 4 is what distinguishes this problem from standard profit maximisation in a monopoly. Usually, the inclusion of higher valuations is assumed to be given, hence the well-established profit maximisation criteria of marginal revenue exceeding marginal cost is applied only when moving from higher valuations to lower ones. In this scenario, the manufacturer has the option to exclude higher indications while including lower ones, hence the criterion needs to be applied going from lower valuations to higher ones as well.

**Corollary 4.** For any $V$ and $x$, there exists a cost structure that results in $P$ being a singleton containing any element of $I$.

*Proof.* It is only profitable to include an indication if the effective cost is below the price. For any indication $i$ with valuation $v_i$ it is never profitable to include an indication $j$ whose valuation is higher as the association cost $\tilde{c}_j$ will be higher than $v_i$. Furthermore, since the set only includes one level of willingness to pay, the manufacturer can set that as the price and capture the entire surplus. □

**Corollary 5.** It is not always possible to construct $c$ such that any subset of $I$ is $P$.

*Proof.* While $\tilde{c}$ can always be increased or reduced to incorporate higher $v_i$'s than the monopoly price, the inverse is not always true. In cases where the revenue loss from incorporating a lower $v_i$ is very large, even a $c_i$ close to zero may not be enough to compensate for it. □

### 5.4.1 Algorithm for Profit Maximising Indication Selection

**Definition 5.7.** Define $H_i = \{k : v_k \geq \tilde{c}_i, i \geq k, i, k \in I\}$ as the set of indications whose value is above the per unit cost of indication $i$.

One obvious way to find the maximum profit is to use results from Lemma 4 and Theorem 7 and check the $\pi_i$ for all $H_i$ and compare. This would result in $|I|$ comparisons.We can refer to this as the Brute Force algorithm

**Lemma 6.** The Brute Force algorithm has a time complexity of $O(|I|^2)$.

*Proof.* In appendix. □

Another way is to use the following algorithm. While it seems to have the same complexity as Brute Force, it provides some more insight into the intuition behind the profit maximising set.

Further, define $H^r = \{H_i : H_i \not\subseteq H_j, H_j \in H^r\}$ so that $H^r$ is a set of $H_i$'s but does not include any elements that are subsets of others.

1 Set $l$ equal to smallest value for which $H_l$ exists in $H^r$.

2 Set $out_l = H_l$

3 If $|H_l| = 1$ set $l$ to the next value for which $H_l$ exists in $H^r$ and go to Step 2. Otherwise, go to Step 4.

4 Set $low$ equal to smallest element in $H_l$ and $up$ equal to the second smallest element in $H_l$

5 If $v_{low} - (v_{up} - v_{low})\frac{\sum_{\{k:k\geq up, k\in H_l\}} x_k}{\sum_{\{k:k\geq low, k<up, k\in H_l\}} x_k} < \frac{\sum_{\{k:k\geq low, k<up, k\in H_l\}} c_k}{\sum_{\{k:k\geq low, k<up, k\in H_l\}} x_k}$,

- remove all elements $\{d : d < up, d \geq low\}$ from $out_l$.
- set $low = up$

6 Set $up$ equal to the next largest value in $H_l$ and repeat Steps 5 and 6 until there is no higher value to move for $up$ in Step 6.

7 Set $l$ to the next value for which $H_l$ exists in $H^r$ and go to Step 2 until the last $H_l$ can be processed.

8 $out_l$'s are now the candidate sets for a profit maximising output. For each one calculate $\pi_l = \underline{v}_l \sum_{\{k:k\in out_l\}} x_k - \sum_{\{k:k\in out_l\}} c_k$.

9 Remove all $out_l$ that exist as subsets of other $out_l$. From the remaining sets, choose $out_l$ which has the highest $\pi_l$.

Intuitively, the algorithm relies on an understanding for why an indication may not be included. The traditional profit maximising problem does not allow for exclusion of consumers with higher valuations. As a result, the idea of marginal revenue only features as a phenomenon going from higher values to lower ones.

However, when exclusion is a possibility then we also need to account for marginal revenue going in the opposite direction - from lower to higher. Since we establish from Lemma 4 that the monopoly price in any set $P$ must be the lowest value, then for any valuation $k$ the inclusion of a higher value $k+1$ results in a marginal revenue of $v_k x_{k+1}$ and a effective cost of $\tilde{c}_{k+1}$.

Thus, we start by identifying indications for which the marginal revenue going from lower to higher values is always higher than the effective cost.

Then, we do the profit maximisation for ironed values as follows. Within each of these sets, we split up the indications into two further sets and charge the lowest price in each, starting by having only the lowest indication in the lower set and everything else in the higher. If the change in revenue going from higher to lower is less than the cost, we drop the lower set and move an indication from the higher to the lower set and repeat the procedure. Otherwise, we move one indication from the higher set to the lower one while keeping the original one and repeat the procedure until we are left with the situation where it is either strictly better to move from charging the highest valuation in the higher set to charging the lowest price in the lowest set or the higher set contains the sole profit maximising indication.

### 5.4.2 Welfare

We now turn towards the basic welfare outcomes in terms of what the surplus looks like for both manufacturer and consumers. In all cases, we stick with the constraints in Theorem 3 hence the results on consumer and producer surplus hold.

From the conditions identified in Theorem 4 we note that regulation introduces trade on previously unviable indications that could be categorised into two groups. Firstly, it introduces indications that would have previously not been included because their value was less than monopoly profit maximising price in a uniform price setting. Secondly, it introduces indications whose costs were too high to justify sale. By including all indications in $M^*$ the surplus from these sales is realised. Thus sales are realised on indications in $M^*\P$ as well as $P$. These include sales from indications in $I$ and $I^f$.

This increase goes completely to the consumer.

**Lemma 7.** The manufacturer's revenue as a result of segmentation must be $v^* = \underline{v}_P \sum_{\{i\in P\}} x_i + \sum_{\{i\in M^*\setminus P\}} c_i$.

*Proof.* Follows mathematically from the fact that $\pi_P = \underline{v}_P \sum_{i\in P} x_i - \sum_{i\in P} c_i$. For inclusion of indications in $M^*$, we have $\pi^*_{M^*} = v^* - \sum_{i\in M^*} c_i$. At the solution, these two must be equal. □

This essentially establishes the principle on which the regulator decides how much the manufacturer must be compensated for a drug. By running the profit maximising algorithm in section 5.4.1, the regulator establishes if an indication would be in the profit maximising set of indications in an unsegmented market. If yes, then the manufacturer must receive the equivalent of what would have been been the uniform monopoly price for each patient with that indication. On the other hand, if the indication would not be included in $P$, then the manufacturer must be compensated by covering the testing costs for that indication.

Thus, intuitively, the regulator assures the manufacturer the profit it would get from uniform pricing and then pays the cost of inclusion for all other indications it wants. For $M^* = I$, this is just all indications that increase total surplus but are not in the profit maximising set. For others it means paying the cost of indications in $I^f$ as long as total surplus does not decrease to zero.

**Corollary 6.** Total consumer surplus, $u^*$ for any $M^*$ can be expressed as

$$u^* = \sum_{\{i\in P\}} (v_i - \underline{v}_P)x_i + \sum_{\{i\in M^*\setminus P\}} (v_i x_i - c_i) + \frac{\delta}{1-\delta} \sum_{\{i\in P\}} v_i x_i$$

*Proof.* Follows obviously from efficiency in Theorem 1 and Lemma 7 □

We can break this result down into two parts. Note that in principle consumers get the surplus from valuations above uniform monopoly price for indications in $P$ and get total surplus for any indications not in $P$. The obvious gain in consumer surplus from segmentation is the entire surplus from indications not sold in an unregulated monopoly.

Note that the indications in $I^f \cap M^*$ contribute negatively to the second term in the expression but have an overall positive impact on $u^*$.

### 5.5 Segmentation Algorithm

I now present an algorithm that can achieve the aim of segmentation that results in the outcomes we have discussed so far.

The regulator starts with receiving $M$ from the manufacturer. Define $I_M = \{i | v_i x_i \geq c_i, i \in M\}$, $I^f = \{i | v_i x_i < c_i \leq \frac{v_i x_i}{1-\delta}, i \in M\}$ and $I_M^c = \{i | \frac{v_i x_i}{1-\delta} < c_i, i \in M\}$

Regulator's declare beforehand what their priorities are and commit to the following algorithm.

1 Calculate $P_M$ using one of the algorithms in section 5.4.

2 Set $M_M^* = I_M \cup I_M^f$.

3 Calculate $v^* = \underline{v}_{P_M} \sum_{\{i \in P_M\}} x_i + \sum_{\{i \in M^* \setminus P_M\}} c_i$

4 Initialise segmentation by setting all $t_{ii} = 1 \forall i$, (i.e. $t_{11} = 1, t_{22} = 1..$) and $t_{ij} = 0 \forall j \neq i$ (i.e. $t_{12} = 0, t_{13} = 0, ..$).

5 For all indications in $M_M^*$, do

   i. Set $l = \min(M_M^*)$.
   ii. Increase all $t_{lk} \forall k > l$ while optimal monopoly price in $s_l$ remains $v_l$ and revenue remains higher than $v^*$, i.e. $\sum_{\{j \geq i | i, j \in M_M^*\}} v_j t_{ji} x_i > v^*$.
   iii. Update $t_{ii} = 1 - \sum_{\{j < i, i \in M_M^*\}} t_{ji}$.
   iv. Set $l$ to next item in $M_M^*$ and go to ii.
   v. Stop when $\sum_{\{j \geq i | i, j \in M_M^*\}} v_j t_{ji} x_i \leq v^*$.

Since the regulator does not put in any money to the market, the maximum it can do is allow first degree price discrimination. Knowing this, the manufacturer knows that the maximum achievable revenue is $\sum_{i \in I} v_i x_i$. Thus the condition $\sum_{i \in M^*} (v_i x_i - c_i) \geq \pi_{P_M}$ is automatically met by manufacturers not exceeding this limit when revealing $M$.

The result is that the manufacturer is able to get the revenue required for $M_M^*$ from those consumers within those indications.

Any indications in $M$ but not in $M*_M$ are set up as separate segments for first degree price discrimination. The return from maximum pricing in these segments does not justify the cost, so the manufacturer should not include any indications that violate the requirements of the regulator.

In this model I take the assumption that the access prioritising regulator looks to include all indications up to the feasible limit. In a real world situation, regulators can set criteria or lists of conditions that would be prioritised within $M^*_M$. Thus the regulator may commit at the start to including only certain indications in $M^*_M$ given that they are feasible.

**Example**

We now look at the how the regulator reacts to receiving $M = \{1, 2, 3, 4\}$ from the example in the introduction.

If $\delta < 0.2$, split up $M$ into $I_M = \{1, 2, 3\}$ and $I^c_M = \{4\}$ and calculate $P = \{2\}$.

The regulator then has $M^*_M = I_M = \{1, 2, 3\}$ thus $v^* = 112 + 40 = \$152$. It hence gives the following segmentation which leads to the highest possible consumer surplus, \$22.

| Segment | Ind 1 | Ind 2 | Ind 3 | Ind 4 | Total | Profit |
|---|---|---|---|---|---|---|
| 1 | 35 | 16 | 3 | 0 | 54 | 54 |
| 2 | 0 | 40 | 0 | 0 | 40 | 80 |
| 3 | 0 | 0 | 6 | 0 | 6 | 18 |
| 4 | 0 | 0 | 0 | 4 | 4 | 16 |
| Total | 35 | 56 | 9 | 4 | 104 | 168 - 80=88 |

Table 5: Consumer Surplus Maximising Segmentation

Knowing this, the manufacturer should omit $\{4\}$ and get the segmentation from the initial example which gives them \$92. Thus the manufacturer is thus dissuaded from including indications from $I^c$.

If $\delta > 0.2$ then $I^f_M = \{4\}$ and $I^c = \varnothing$. The regulator sets $M^*_M = I_M$, and given that $(35\times1)+(56\times2)+(9\times3)+(4\times4)-(4\times20) = 110 > 92$ it adds $\{4\}$ leading to $M^*_M = \{1, 2, 3, 4\}$.

Then $v^* = 92 + 80 = \$172$ and the segmentation is.

| Segment | Ind 1 | Ind 2 | Ind 3 | Ind 4 | Total | Profit |
|---|---|---|---|---|---|---|
| 1 | 35 | 19 | 3 | 1 | 58 | 58 |
| 2 | 0 | 37 | 0 | 0 | 37 | 74 |
| 3 | 0 | 0 | 6 | 0 | 6 | 18 |
| 4 | 0 | 0 | 0 | 3 | 3 | 12 |
| Total | 35 | 56 | 9 | 4 | 104 | 172 - 80=92 |

Table 6: Consumer Surplus Maximising Segmentation

Table 7 another possible solution for maximising consumer surplus, highlighting the non-uniqueness of the problem. In both cases, the consumer surplus accrues to different segments of the consumer base.

| Segment | Share 1 | Share 2 | Share 3 | Total | Profit |
|---|---|---|---|---|---|
| 1 | 35 | 4 | 9 | 48 | 48 |
| 2 | 0 | 52 | 0 | 52 | 104 |
| Total | 35 | 56 | 9 | 100 | 152 - 60=92 |

Table 7: 3-Indication Example - Alternate Segmentation Algorithm

As stated earlier, this is one of a set of feasible solutions. For $m > 2$ as long as there is non-zero consumer surplus in the first period (i.e. $\sum_{i \in M}(v_i x_i - c_i) > \pi_P$), there can exist different ways of splitting it among consumers. I explore two different considerations for consumer surplus division in section 5.7.

## 5.6 Output

We can assume the case of a regulator who is more concerned with maximising consumer health than surplus. Then their utility function becomes

$$u^* = \max_{\{t_{ji}^M | i,j \in M\}} \sum_{i \in M} \sum_{j | j \leq i \in M} t_{ji} v_i x_i + \frac{\delta}{1-\delta} \sum_{i \in M} v_i x_i$$

In these cases, the regulator essentially uses surplus from indications in $I$ to pay for inclusion of indications from $I^c$. Essentially the same principle applied as did with the inclusion of indications from $I^f$ in that surplus is reinvested to maximise trade in the first period. Once again,, we hit an upper bar on how many of these indications can be included when $\sum_{i \in M^*}(v_i x_i - c_i) = \pi_P^U$.

Simplifying even further we could just be concerned with pure output

$$u^* = \max_{\{t_{ji}^M | i,j \in M\}} \sum_{i \in M} \sum_{j | j \leq i \in M} t_{ji} x_i + \frac{\delta}{1-\delta} \sum_{i \in M} x_i$$

While the principle of reinvesting surplus remains the same as before our choice of indications to include outside of $I$ will be driven by different factors i.e. those that maximise output or health rather than long run consumer surplus. The regulator cannot force the manufacturer to make the optimal choice in either case, however, since the manufacturer is made indifferent between both cases on may assume a costless signal from the regulator is enough.

To achieve this, the algorithm in section 5.5 can be adapted by simply changing the second step to

2 Set $M_M^* = M$.

and keeping everything else the same.

However, we do note that there can now be a conflict between consumer surplus maximisation or output maximisation based on whether indications from $I^c$ are included in $M$.

**Lemma 8.** If $I^c = \varnothing$ then the solution to the regulator's problem is the same regardless of priorities. If $I \subset D$, then there is a tradeoff between access and consumer surplus.

*Proof.* The maximum total surplus is given by $\sum_{i \in M^*}(v_i x_i - c_i)$. Consumer surplus is what remains after the $\pi_{M^*}^*$ satisfies the inclusion constraint. The non-negativity constraint ensures that all indications in $M^*$ are accessible. For $D = I$, the inclusion of all indications weakly improves consumer surplus, as show in lemma 3. Thus, trade of all $I$ increases consumer surplus and access. For access, inclusion of indications in $I^c$ can be done on top of $I$. They reduce consumer surplus, but the additional indications translate to increased access. □

### 5.7 Surplus distribution considerations

From Theorem 3 we have established that fixing the manufacturer's profit and ensuring efficiency means that maximising consumer surplus is guaranteed. We also, know that in a majority of cases the feasible set of solutions is not a single-ton. This means that we can focus on other considerations in our choice of solution.

We focus primarily on the division of surplus among consumers in the first period. We focus our attention on distribution of consumer surplus in the monopoly period because prices are driven by costs rather than consumer valuations in the competitive market so segmentation has no impact.

An obvious consideration on distribution might be to keep the benefits of segmentation evenly spread across consumers. Price dispersion would be a natural candidate to minimise given that it result in minimising differences in utility among consumers. Another candidate might be the dispersion in consumer surplus, as that captures the gain in utility among consumers as a result of using the drug.

Thus, I look at the variance minimisation in price paid in section 5.7 and variance minimisation in consumer surplus in section 5.7 over the population of consumers who purchase the drug. I present solutions for the case with three indications and identify the general principles that they highlight. Before doing so, I present some preliminaries of the problem.

Since we deal with a unit population, the probability of a consumer of indication $i$ in segment $j$ is simply given by $t_{ji}x_i$ given that the consumer has indication in $M^*$. The expected price paid is the same as expected revenue is then given by

$$v^* = \sum_{i \in M^*} \sum_{j \leq i \in M^*} \frac{t_{ji}x_i}{\sum_{i \in M^*} x_i} v_j$$

.

while consumer surplus in the monopoly period is given by

$$u_0^* = \sum_{i \in M^*} \sum_{j \leq i \in M^*} \frac{t_{ji}x_i}{\sum_{i \in M^*} x_i} (v_i - v_j)$$

Thus, the problem of minimising dispersion amounts to solving the following optimisation problem. We look to minimize $U()$ which represent the variance of price paid and consumer surplus respectively:

$$U_1() = \sum_{i \in M^*} \sum_{j \leq i \in M^*} \frac{t_{ji}x_i}{\sum_{i \in M^*} x_i} v_j^2 - \left[ \sum_{i \in M^*} \sum_{j \leq i \in M^*} \frac{t_{ji}x_i}{\sum_{i \in M^*} x_i} v_j \right]^2$$

$$U_2() = \sum_{i \in M^*} \sum_{j \leq i \in M^*} \frac{t_{ji}x_i}{\sum_{i \in M^*} x_i} (v_i - v_j)^2 - \left[ \sum_{i \in M^*} \sum_{j \leq i \in M^*} \frac{t_{ji}x_i}{\sum_{i \in M^*} x_i} (v_i - v_j) \right]^2$$

under the constraints presented in Theorem 3.

The expression in the brackets in both cases is a constant by definition, $u_0^*$ and $v^*$ respectively. This means that for both case, the objective function becomes linear and the problem is a linear program.

$$v^* = \sum_i \sum_{j \leq i} t_{ji} x_i v_j = \sum_i v_i x_i - \sum_i \sum_{j<i} (v_i - v_j) t_{ji} x_i = \sum_i v_i x_i - u^*$$

and

$$\sum_i \sum_{j<i} t_{ji} x_i (v_i - v_j) = u_0^*$$

To clarify the link between $v^*$ and $u_0^*$ mentioned in the constraints above, let us state it explicitly, i.e. $v^* = w_0^* + \sum_{i \in I} c_i - u_0^*$, where $w_0 = \sum_{i \in I}(v_i x_i - c_i)$.

Given that all sunk costs, we note that there is a significance to the structure of the market within $M^*$ without costs.

**Definition 5.8.** Let $v'$ be the profit maximising price for the market without costs.

### Distributional Considerations given Three Indications

As stated earlier, we are dealing with linear programs for both price and consumer surplus variance minimisation. The case of three indications in $M = M^*$ is more tractable given that we have established in Theorem 3 that it involves 2 choice variables and linear constraints. I present results for each problem for $m = 3$, and highlight where these results are generalisable to any $m$.

For simpler notation we assume that $D = I = M^*$, but this assumption is not necessary for the results. Assuming there are no excluded patients from the $D$ means we do not have to rescale probabilities as done so in the previous section. Furthermore, having indications only from $I$ ensures that there is always a positive consumer surplus to distribute hence we need not make any qualifications on that end. The solutions will hold for any $M = M^*$ where there are three indications and a positive consumer surplus to be distributed in the first period.

This problem is simpler to solve because, given $v^*$, it is a problem in two free variables. This is because we have three free variables, $t_{12}, t_{13}, t_{23}$, without the revelation constraint. Since the revelation constraint is an equality, $t_{23}$ can be expressed in terms of $t_{12}, t_{13}$. Hence we make the following substitution into our extant constraint set:

$$t_{23} = \frac{u_0^* - t_{12}(v_2 - v_1)x_2 - t_{13}(v_3 - v_1)x_3}{(v_3 - v_2)x_3}$$

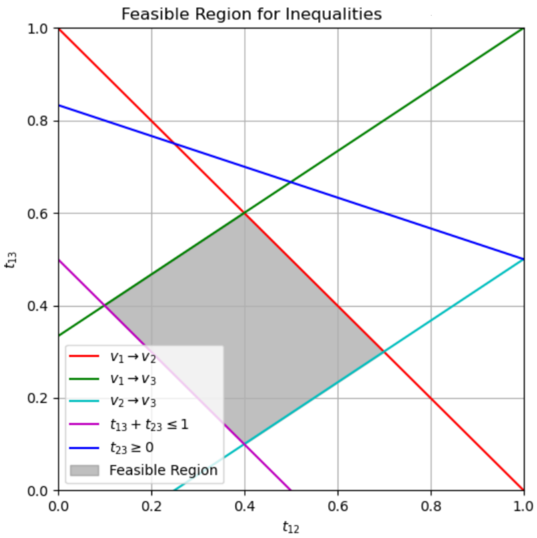
Each expression can be written in terms of $u_0^*$ or $v^*$ where $u_0^* = \sum_i v_i x_i - v^*$. Thus,

$$U_1 = (v_3 - v_2)u_0^* - [u_0^*]^2 + (v_2 - v_1)(2v_2 - v_1 - v_3)x_2 t_{12} + (v_3 - v_1)(v_2 - v_1)x_3 t_{13}$$

$$U_2 = v_1^2 x_1 + v_2^2 x_2 + v_3^2 x_3 - (v_3 + v_2)u_0^* - [v_1 x_1 + v_2 x_2 + v_3 x_3 - u_0^*]^2 + (v_2 - v_1)(v_3 - v_1)[t_{12} x_2 + t_{13} x_3]$$

,

Let $P \subseteq \{1, 2, 3\}$ be the profit maximising subset, as defined earlier, then the profit that must be achieved is $\pi_P = \underline{v}_P \sum_{i \in P} x_i - \sum_{i \in P} c_i$. This means that the mean price (revenue)




$v^* = \underline{v}_P \sum_{i \in P} x_i + \sum_{i \notin P} c_i$. Hence we know that the segmentation must deliver this as the expected price.

Suppose $P = \{1, 2, 3\}$, i.e. it is profit maximising to include all indications because the set satisfies the criteria in Theorem 3. Then there is nothing left for the regulator to do. There is only one segment which is charged a price of $v_1$ resulting in $v^* = v_1$ and a variance of zero.

If $P = \{1, 2\}$ then we know that is must be because $c_3 > v_1 x_3$. Consequently we know that $v^* = v_1(x_1 + x_2) + c_3 > v_1$. Similarly if $P = \{2, 3\}$ then it must be that $c_1 > v_1 x_1 - (v_2 - v_1)(x_2 + x_3)$ and again $v^* = v_2(x_2 + x_3) + c_1 > v_1$. Another way of looking at this is to say that since we have a non-negativity constraint, we know that $v^*$ in all situations is at least greater than $v_1(x_1 + x_2 + x_3) = v_1$. Furthermore, it means that there is always a segment $s_1$.

Given that $v^*$ is established from a somewhat exogenous process through identifying $P$, we now proceed with that as an input. I present the process of minimising variance in price, $U_1$, first as it provides the simplest cases and formulates good intuition which can be used for the case of $U_2$.

The first overarching result is that if $v^*$ can be achieved within two segments, then the variance minimising outcome must do so. Furthermore, if $v^*$ is not achievable within two segments then the first segment must only contain patients with indication 1 i.e. $t_{12} = t_{13} = 0$.

A detailed graphical proof is provided in the appendix, however, for now suffice to say that $U_1$ is minimised on the line showing the constraint $t_{13} + t_{23} = 1$ in the figure above. If this line is does not appear in the first quadrant, then the optimal solution is to set $t_{12}, t_{13} = 0$.

So for now, it is worth looking at this case by case to highlight under what circumstances this constraint is able to bind.

The following general results provide some insight into this.

**Lemma 9.** For any $m$, $s_k$ must exist if $v^* > \sum_{i=1}^{k-2} v_i x_i + v_{k-1} \sum_{i \geq k-1}^{m} x_i$.

*Proof.* The required expected revenue is not achievable by even putting everyone else in the segment just under $k$. □

**Lemma 10.** For any $m$, $s_z$ must exist if $v^* < \sum_{i=1}^{k-2} v_i x_i + v_{k-1} \sum_{i \geq k-1}^{m} x_i$ and one there is some $v_z$ where $z > k$ such that $v_{k-1} \sum_{i \geq k-1}^{m} x_i < v_z \sum_{i \geq z}^{m} x_i$.

*Proof.* Obedience constraint violated may also lead to it. Recheck. □

**Theorem 5.** For any $m$, then the variance minimising solution for any $v^* < \sum_{i=1}^{M-1} v_i x_i + v' \sum_{i \geq M}^{n} x_i$ requires the non-existence of $s_j, \forall j > M$.

*Proof.* We know that $v^*$ is achievable within first $M$ segments. Furthermore, we know that $\sum_{i=1}^{M-1} v_i xi + v_M \sum_{i \geq M}^{I} x_i$ is achievable within the obedience constraints and without $s_j, \forall j > M$. Since we know that $v^* < v_M$, charging any patients a value higher than $v_M$ will obviously lead to more of an increase in variance than charging them $v_M$. Hence, a variance minimising outcome must exclude $s_j, \forall j > M$. □

Let's look at the implications of these results for $m = 3$.

**The Role of $t_{12}$:** Before proceeding to the rest of the section, it might be useful to understand the basic implications of the constraints in Theorem 3. The choice of $t_{12}$ provides insight into how we may go about achieving a certain solution.

Increasing $t_{12}$ has two impacts. Firstly, the direct and obvious impact is that it increases the number of indication 2 patients in $s_1$. This results in increased consumer surplus being achieved through these patients.

The second, indirect effect is that it reduces the number of indication 3 patients that can be fit into $s_2$ because it has reduced the number of indication 2 patients in that group. Hence, it may end up reducing the consumer surplus attained in $s_2$ because $t_{23}$ may have to be reduced in order to satisfy the obedience constraint for $s_2$.

We can generalize this in saying that any increase in $t_{ji}$, could result in a decrease in consumer surplus in $s_i$. Having understood this, we proceed to the solutions.

Let's start by defining $\bar{v} = v_1(x_1 + x_3) + v_2 x_2 \left( \frac{v_3 - v_1}{v_3 - v_2} \right)$. This is the total revenue that can be generated if $t_{12} = 1$ without $s_3$ and the obedience constraint in $s_2$ binds.

**Corollary 7.** For $n = 3$, $s_3$ will always exist if

1. $v^* > v_1 x_1 + v_2(x_2 + x_3)$ , and/or
2. $\bar{v} < v^* < v_1 x_1 + v_3 x_3$.

*Proof.* The first part is self-evident. The required $v^*$ is too high to be achieved without charging some patients a value of $v_3$. The second part is a result of the binding constraint that keeps the monopoly price in $s_2$ at $v_2$. This can only happen if $v' = v_3$, i.e. $v_3 x_3 > v_2(x_2 + x_3)$. There are too few indication 2 patients to accommodate all the indication 3 patients in $s_2$. □

So we understand the role the underlying market structure plays in the revenues that are achievable within a certain number of sets. Essentially, we know that for any $v_i = v'$ if all the patients of $i$ are put in segment $s_i$ then no obedience constraint will bind in that segment. This is intuitive from the definition of $v'$, as there is no other profit maximising price.

Thus we can proceed to answer the question of price variance minimisation by splitting up the results into two different case: $v' = v_2$ and $v' = v_3$. Furthermore, henceforth, I denote each obedience constraint in terms of $v_j \to v_k$ where

$$\sum_{j<i<k} t_{ji} v_j x_i \geq \sum_{i \geq k} t_{ji}(v_k - v_j)x_i \quad \forall k, j \in M^* \; s.t. \; k > j$$

So $v_j \to v_k$ denotes the obedience constraint keeping the profit maximising price in $s_j$ equal to $v_j$ rather than $v_k$. When this constraint binds, we know that either $v_j$ or $v_k$ could serve as profit maximising price in $s_j$, and assume that manufacturer opts for $v_j$.

#### Price Variance Minimisation

We now proceed to the two cases $v' = v_2$ and $v' = v_3$. For the case where $v' = v_2$, we have argued that the constraint $t_{13} + t_{23} = 1$ can be made to bind as long as $v^* < v_1 x_1 + v_2(x_2 + x_3)$. Thus we focus on these situation where this binds, and where it doesn't.

**Case 1:** $v_2(x_2 + x_3) > v_3 x_3$

From the non-negativity constraint, we know that all patients with valuation $v_1$ are in $s_1$. We now find the variance minimising segmentations based on the value of $v^*$ that has a lower bound of $v_1$.

$v^* < v_1 x_1 + v_2(x_2 + x_3)$ : No patients are put in segment $s_3$, but are divided between $s_1$ and $s_2$. There are an infinite number of possible segmentations which satisfy $t_{13} + t_{23} = 1$, where $t_{12} \leq \frac{(v_3 - v_2)(\bar{v} - v^*)}{v_3 x_2 (v_2 - v_1)}$ and $t_{13} \geq 1 - \frac{v_2(v^* - v_1)}{v_3 x_3 (v_2 - v_1)}$, and $t_{12} \geq \max\{\frac{v_1(x_1 + x_3) + v_2 x_2 - v^*}{(v_2 - v_1)x_3}, \frac{v_3 - v_1}{v_2 - v_1}\frac{1}{v_2 x_2}(v_1 x_1 + v_2(x_2 + x_3) - v^*) - \frac{v_1 x_1}{v_2 x_2}, 0\}$ and reciprocal limits on $t_{13}$. The basic point is that $v^*$ is very small and is achievable within two segments; the only consideration is that the obedience constraints hold. In this case, the minimised variance is

$$U_2 = v_1^2 x_1 + v_2^2 x_2 - v_1 v_2 x_3 - (v_1 + v_2)[v_1 x_1 + v_2 x_2 - v^*] - {v^*}^2$$

$$U_2 = (v_1 + v_2)v^* - {v^*}^2 - v_1 v_2$$

$v_1 x_1 + v_2(x_2 + x_3) < v^*$ : There is now no option but to create $s_3$ as $v^*$ is not achievable even by putting everyone with a valuation above $v_2$ in $s_2$. We now achieve a unique solution which nobody other than $v_1$ is placed in $v_1$. The entire consumer surplus requirement is then achieved through the patients of type 3 being put in $s_2$. Thus, the fraction $t_{23} = \frac{u_0^*}{(v_3 - v_2)x_3}$ of indication 3 patients are placed in $s_2$. Since $v_2(x_2 + x_3) > v_3 x_3$ any value of $t_{13}$ does not violate the obedience constraint in $s_2$.

In this case, the minimised variance is

$$U_2 = v_1^2 x_1 + v_2^2 x_2 + v_3^2 x_3 - (v_3 + v_2)[v_1 x_1 + v_2 x_2 + v_3 x_3 - v^*] - {v^*}^2$$

$$U_2 = (v_2 + v_3)v^* - {v^*}^2 - v_1 x_1 (v_2 - v_1) - v_3 x_3 (v_1 + v_2)$$

**Case 2:** $v_3 x_3 > v_2(x_2 + x_3)$

We now move to the case where $v_3 x_3 > v_2(x_2 + x_3)$ hence the obedience constraint in $s_2$ plays a more important role. This is because there just aren't enough patients with indication 2 to offset the profit from indication 3. Thus, if all patients with valuation $v_1$ are placed in $s_1$ and the remaining are put in another group, then the monopolist would simply charge $v_3$ in the other group, hence eliminating $s_2$ and introducing $s_3$. Therefore, is it important to not put too many patients with indication 3 in $s_2$.

$v^* < \bar{v}$ : This is the only situation in this Case where there is no $s_3$. Lowest variance can be achieved through splitting indication 3 patients between $s_1$ and $s_2$. This is identical to the case of $v^* < v_1 x_1 + v_2(x_2 + x_3)$ when $v' = v_2$ as described above.

$\bar{v} < v^* < v_1 x_1 + v_3 x_3$ : As discussed earlier, this range requires the creation of $s_3$. We deal with this by firstly ensuring that all indication 2 patients are in $s_2$ and then further by splitting indication three between $s_1, s_2$ and $s_3$. The unique solution here is $t_{13} = \frac{u_0^* - v_2 x_2}{(v_3 - v_1)x_3}$, $t_{23} = \frac{v_2 x_2}{(v_3 - v_1)x_3}$ and putting the residuals in $s_3$, $t_{33} = 1 - t_{13} - t_{23}$.

$v_1 x_1 + v_3 x_3 < v^*$ : Here again, we get a unique solution identical to the last situation in Case 1. The high $v^*$ requires that a larger proportion of patients are charged $v_3$ by putting them in $s_3$. Simultaneously, the requirement to reduce variance means that we use $s_2$ to make up for the consumer surplus requirement. Hence, $t_{23} = \frac{u_0^*}{(v_3 - v_2)x_3}$.

**Lemma 11.** For $m = 3$,

1 if $v' = v_2$ and $v^* > v_1 x_1 + v_2(x_2 + x_3)$ or

2 if $v' = v_2$ and $v^* > \bar{v}$

then the variance minimising segmentation has $t_{12} = 0$.

*Proof.* If $v^*$ is higher than what can be accommodated within 2 segments, then variance minimisation would demand that patients be put in $s_2$ rather than $s_1$. So to ensure consumer surplus, the first priority is to move indication 3 patients into $s_2$. In order to do that $t_{12}$ is maximised to ensure the maximum number of indication 3 patients in $s_2$. □

### Consumer Surplus Variance Minimisation

We now look at the solution that minimises the variance in consumer surplus. As opposed to the previous situation, consumer surplus depends on the relative values of prices rather than absolute prices. As such, we find three situations which are dealt with separately.

Firstly, we look at $U_1$ and note that there are distinct cases based on the relation between $v_3 - v_2$ and $v_2 - v_1$, which can also be expressed as the relation between $v_2$ and the average of $v_1$ and $v_3$. Within $v_2 < \frac{v_3 + v_1}{2}$ we can draw two further distinctions between the case where $v_2 > \frac{v_3}{2}$ and $v_2 < \frac{v_3}{2}$. Hence, we get different outcomes based on where $v_2$ places among the two thresholds $\frac{v_3 + v_1}{2}$ and $\frac{v_3}{2}$.

The primary reasoning in this section relies on the fact that $v_2 > \frac{v_3+v_1}{2}$ implies that $v_2 - v_1 > v_3 - v_2$. This inequality holding implies that there is a greater increase in consumer surplus from putting indication 2 patients in $s_1$ than from putting indication 3 patients in $s_2$. If we are looking to minimise variance in consumer surplus, we look to spread the surplus over a larger number of patients, hence we prefer putting indication 3 patients in $s_2$ over putting indication 2 patients in $s_1$.

Similarly, for the inverse, we prefer to fill $s_1$ with indication 2 patients over putting indication 3 patients in $s_2$. Both are more desirable to putting indication 3 patients in $s_1$, however, in the case that $v_2 < \frac{v_3}{2}$, the tradeoff between adding indication 2 patients in $s_1$ to adding indication 3 patients to $s_2$ is so high that it is preferable to put indication 3 patients in $s_1$ rather than moving any indication 2 patients from $s_1$ to $s_2$.

The consumer surplus variance minimising solution can be gotten to directly, but I present algorithms for each situation first as it provides intuition for the solution. I give the solutions separately after the algorithms. Furthermore, the algorithm for the case where $v_2 > \frac{v_3+v_1}{2}$, this algorithm also provides a solution for price variance minimisation.

Recall the definition of $t_{ji}$ is the share of indication $i$ patients in $s_j$. This implies that increasing (decreasing) $t_{ji}$ means adding (removing) indication $i$ patients to (from) $s_j$. Increasing $t_{ji}$ by decreasing $t_{ki}$ implies moving indication $i$ patients from $s_k$ to $s_j$. Recall further that we are interested in $t_{ji}$ where $j < i$.

In all cases: Initialize $t_{11} = t_{22} = t_{33} = 1$, all other $t$'s set to zero i.e. we start at first degree price discrimination with zero consumer surplus and start moving patients into different segments, gradually increasing consumer surplus. The brackets after each step show the outcome if the step is completed, and consequently, the maximum $u_0^*$ achievable in that step.

**For** $v_2 > \frac{v_3+v_1}{2}$ :

If $v_2(x_2 + x_3) > v_3x_3$:

- While $u < u_0^*$, do:

  1 Increase $t_{23}$ by reducing $t_{33}$ until $t_{33} = 0$
  $[u_0^* = (v_3 - v_2)x_3]$ $[t_{12} = 0, t_{13} = 0, t_{23} = 1]$

  2 Increase $t_{12}$ by reducing $t_{22}$ until obedience constraint $v_2 \to v_3$ binds
  $\left[u_0^* = (v_2 - v_1)x_2 + \frac{v_1}{v_2}(v_3 - v_2)x_3\right]$ $\left[t_{12} = \frac{v_2(x_2+x_3)-v_3x_3}{v_2x_2}, t_{13} = 0, t_{23} = 1\right]$

  3 Increase $t_{13}$ and $t_{22}$, by reducing $t_{12}$ and $t_{23}$, always ensuring that $v_2 \to v_3$ binds.

If $v_3x_3 > v_2(x_2 + x_3)$:

- While $u < u_0^*$, do:

  1 Increase $t_{23}$ by reducing $t_{33}$ until $v_2 \to v_3$ binds
  $[u_0^* = v_2x_2]$ $\left[t_{12} = 0, t_{13} = 0, t_{23} = \frac{v_2x_2}{(v_3-v_2)x_3}\right]$

  2 Increase $t_{13}$ by reducing $t_{33}$, until $t_{33} = 0$
  $\left[u_0^* = v_2x_2\left(\frac{v_2-v_1}{v_3-v_2}\right) + (v_3 - v_1)x_3\right]$ $\left[t_{12} = 0, t_{13} = \frac{v_3x_3-v_2(x_2+x_3)}{(v_3-v_2)x_3}, t_{23} = \frac{v_2x_2}{(v_3-v_2)x_3}\right]$

  3 If $t_{33} = 0$, increase $t_{12}$ and $t_{13}$ by reducing $t_{22}$ and $t_{33}$.

**For** $\frac{v_3}{2} < v_2 < \frac{v_3+v_1}{2}$ :

If $v_1(x_1 + x_2) > v_2x_2$:

- While $u < u_0^*$, do:

  1 Increase $t_{12}$ by reducing $t_{22}$ until $t_{22} = 0$.
  $[u_0^* = (v_2 - v_1)x_2]$ $[t_{12} = 1, t_{13} = 0, t_{23} = 0]$

  2 Increase $t_{22}$ and $t_{23}$ by reducing $t_{12}$ and $t_{33}$ making sure $v_2 \to v_3$ binds until (i) [if $v_2(x_2 + x_3) > v_3x_3$] $t_{33} = 0$ or (ii) [if $v_3x_3 > v_2(x_2 + x_3)$] $t_{22} = 1$
  (i) $\left[u_0^* = (v_2 - v_1)x_2 + \frac{v_1}{v_2}(v_3 - v_2)x_3\right]$ $\left[t_{12} = \frac{v_2(x_2+x_3)-v_3x_3}{v_2x_2}, t_{13} = 0, t_{23} = 1\right]$
  (ii) $[u_0^* = v_2x_2]$ $\left[t_{12} = 0, t_{13} = 0, t_{23} = \frac{v_2x_2}{(v_3-v_2)x_3}\right]$

  3 If $v_3x_3 > v_2(x_2 + x_3)$, increase $t_{13}$, by reducing $t_{33}$ until $t_{33} = 0$.
  $[u_0^* = v_1x_1 + v_2x_2]$ $\left[t_{12} = 0, t_{13} = \frac{v_1x_1}{(v_3-v_1)x_3}, t_{23} = \frac{v_2x_2}{(v_3-v_2)x_3}\right]$

  4 Increase $t_{12}$ and $t_{13}$, by reducing $t_{22}$ and $t_{23}$, always ensuring that $v_2 \to v_3$ binds.

If $v_2x_2 > v_1(x_1 + x_2)$:

- While $u < u_0^*$, do:

  1 Increase $t_{12}$ by reducing $t_{22}$ until $v_1 \to v_2$ binds.
  $[u_0^* = v_1x_1]$ $\left[t_{12} = \frac{v_1x_1}{(v_2-v_1)x_2}, t_{13} = 0, t_{23} = 0\right]$

  2 Increase $t_{23}$ by reducing $t_{33}$ until $v_2 \to v_3$ binds.
  $\left[u_0^* = v_2x_2 - \left(\frac{v_1}{v_2-v_1}\right) v_1x_1\right]$ $\left[t_{12} = \frac{v_1x_1}{(v_2-v_1)x_2}, t_{13} = 0, t_{23} = \left(\frac{v_2}{v_2-v_1}\right)\left(\frac{v_2x_2-v_1(x_1+x_2)}{(v_3-v_2)x_3}\right)\right]$

  3 Increase $t_{22}$ and $t_{23}$ by reducing $t_{12}$ and $t_{33}$ making sure $v_2 \to v_3$ binds until (i) [if $v_2(x_2 + x_3) > v_3x_3$] $t_{33} = 0$ or (ii) [if $v_3x_3 > v_2(x_2 + x_3)$] $t_{22} = 1$
  (i) $\left[u_0^* = (v_2 - v_1)x_2 + \frac{v_1}{v_2}(v_3 - v_2)x_3\right]$ $\left[t_{12} = \frac{v_2(x_2+x_3)-v_3x_3}{v_2x_2}, t_{13} = 0, t_{23} = 1\right]$
  (ii) $[u_0^* = v_2x_2]$ $\left[t_{12} = 0, t_{13} = 0, t_{23} = \frac{v_2x_2}{(v_3-v_2)x_3}\right]$

  4 If $v_3x_3 > v_2(x_2 + x_3)$, increase $t_{13}$, by reducing $t_{33}$ until $t_{33} = 0$.
  $[u_0^* = v_1x_1 + v_2x_2]$ $\left[t_{12} = 0, t_{13} = \frac{v_1x_1}{(v_3-v_1)x_3}, t_{23} = \frac{v_2x_2}{(v_3-v_2)x_3}\right]$

  5 Increase $t_{12}$ and $t_{13}$, by reducing $t_{22}$ and $t_{23}$, always ensuring that $v_2 \to v_3$ binds.

**For** $v_2 < \frac{v_3}{2}$ :

If $v_1(x_1 + x_2) > v_2x_2$:

- While $u < u_0^*$, do:

  1 Increase $t_{12}$ by reducing $t_{22}$ until $t_{22} = 0$.
  $[u_0^* = (v_2 - v_1)x_2]$ $[t_{12} = 1, t_{13} = 0, t_{23} = 0]$

  2 Increase $t_{13}$ by reducing $t_{33}$ until $t_{33} = 0$.

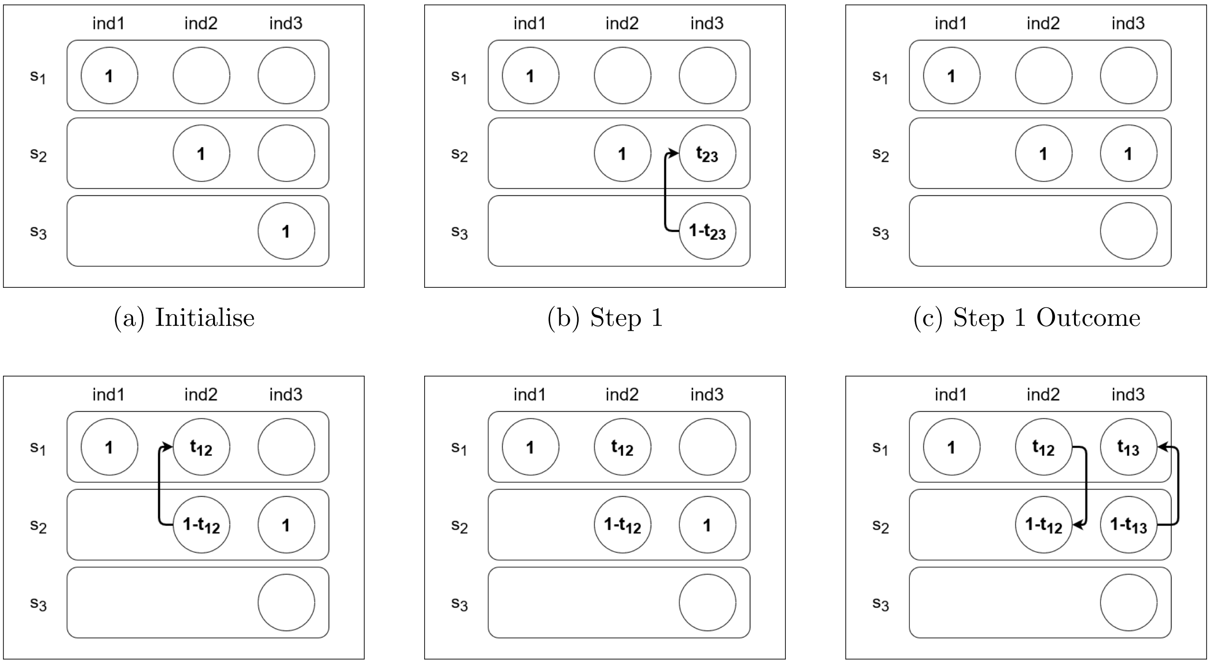
If $v_2x_2 > v_1(x_1 + x_2)$:

- While $u < u_0^*$, do:

1 Increase $t_{12}$ by reducing $t_{22}$ until $v_1 \to v_2$ binds
$[u_0^* = v_1x_1]$ $\left[t_{12} = \frac{v_1x_1}{(v_2-v_1)x_2}, t_{13} = 0, t_{23} = 0\right]$

2 Increase $t_{23}$ by reducing $t_{33}$ until $v_2 \to v_3$ binds or $t_{33} = 0$.
$\left[u_0^* = v_2x_2 - \left(\frac{v_1}{v_2-v_1}\right) v_1x_1\right]$ $\left[t_{12} = \frac{v_1x_1}{(v_2-v_1)x_2}, t_{13} = 0, t_{23} = \left(\frac{v_2}{v_2-v_1}\right)\left(\frac{v_2x_2-v_1(x_1+x_2)}{(v_3-v_2)x_3}\right)\right]$

3 Increase $t_{13}$ and $t_{22}$, by reducing $t_{12}$ and $t_{33}$, always ensuring that $v_1 \to v_2$ and $v_2 \to v_3$ binds.




(a) Initialise (b) Step 1 (c) Step 1 Outcome

(d) Step 2 (e) Step 2 Outcome (f) Final Step

Figure 4: Algorithm for the case where $v_2 > \frac{v_3+v_1}{2}$ and $v_2(x_2 + x_3) > v_3x_3$

Figure 4 shows how the algorithm would look in the case of $v_2 > \frac{v_3+v_1}{2}$ if $v_2(x_2+x_3) > v_3x_3$. The horizontal rectangles indicated the three segments while the circle in each column represent what ratio of each indication is in each segment. Thus, we initialise with each indication in its out segment, such that all diagonal circles have fraction 1. The arrows show movement of patients in an indication between segments.

We now turn to the solution of the algorithm based on required $v^*$. Graphical proofs of these solutions are given in the Appendix. Outcomes are based on valuations, $V$, the distribution of patients,$x$, and the target revenue, $v^*$ (remember that $v^* = v_1x_1 + v_2x_2 + v_3x_3 - u_0^*$).

**Situation 1:** $v_2 > \frac{v_3+v_1}{2}$

We discuss these situations one by one. We start by defining $\bar{\bar{v}} = v_1 + v_3x_3\left(\frac{v_2-v_1}{v_2}\right)$ as the achievable outcome when the obedience constraint in $s_2$ binds.

The solutions in this situation also minimise variance in price, but are unique. As such, they also rely on avoiding the creation of $s_3$.

**Case 1** $v_2(x_2 + x_3) > v_3x_3$**:**

$v^* < v_1x_1 + v_2(x_2 + x_3)$ : Variance in Consumer Surplus is minimised by putting the least amount of indication 3 patients in segment 1 and the most in segment 2. The restriction here is if too many are put in segment 2, then the obedience constraint for that segment might be violated. Hence, if feasible, set $t_{13} = 0$, $t_{23} = 1$ and $t_{12} = \frac{v_1x_1+v_2(x_2+x_3)-v^*}{(v_2-v_1)x_2}$. Thus, the consumer surplus is made up as much as possible by from indication 3 patients in segment 2, and then by indication 2 patients in segment 1.

This is not feasible if the obedience constraint binds ($v^* < \bar{\bar{v}}$) because the consumer surplus $u_0^*$ is not achievable through the solution above. In this case, set $t_{12} = \frac{(v_3-v_2)(\overline{v}-v^*)}{v_3x_2(v_2-v_1)}$ and $t_{13} = 1 - \frac{v_2(v^*-v_1)}{v_3x_3(v_2-v_1)}$. These are the values that minimise the number of indication 3 patients in segment 1 within the feasible set and maximise the number in segment 2.

$v^* > v_1x_1 + v_2(x_2 + x_3)$ : This situation is identical to price variance minimisation. As we know that priority one is to make up the consumer surplus through putting indication 3 patients in $s_2$, and we know that the required consumer surplus is low enough to be fulfilled in this way, we get that $t_{23} = \frac{u_0^*}{(v_3-v_2)x_3}$, while the $t_{12}, t_{13} = 0$.

**Case 2** $v_3x_3 > v_2(x_2 + x_3)$**:**

The only situation where the solution here is different to Case 1 is when $\overline{v} < v^* < v_1x_1 + v_3x_3$, in that case, the solution is identical to the one for price variance minimisation i.e. $t_{13} = \frac{u_0^*-v_2x_2}{(v_3-v_1)x_3}$, $t_{23} = \frac{v_2x_2}{(v_3-v_1)x_3}$ and putting the residuals in $s_3$, $t_{33} = 1 - t_{13} - t_{23}$. In this case, it is not feasible to put any indication 2 patients in $s_1$ as that would mean having to reduce indication 3 patients in $s_2$.

**Situation 2:** $\frac{v_3}{2} < v_2 < \frac{v_3+v_1}{2}$

Our primary objective is to try and achieve the consumer surplus target through putting indication 2 patients in $s_1$. If that is not enough to achieve the target, then some of them have to be moved to $s_2$ to allow for more indication 3 patients in $s_2$.

The situations of what is the monopoly price is less relevant as a divider in this case. The only distinction it allows us to draw is in one particular case mentioned below. In symmetry with the situation where $v_2 > \frac{v_3+v_1}{2}$, try to put as many indication 2 patients in $s_1$ without obedience constraints being violated. There are two obedience constraints that need to be considered. Firstly, putting in too many indication 2 patients in $s_1$ can flip the monopoly price in $s_1$. Alternatively, it draws indication 2 patients from $s_2$ and can cause the monopoly price there to flip to 3. Thus, we increase $t_{12}$ until one of the two constraints binds. The limits can be expressed then in each scenario.

**Case 1:** $v_2(x_2 + x_3) > v_3x_3$

$v^* < \bar{\bar{v}}$ : The required consumer surplus cannot be achieved without putting some patients from indication 3 in $s_1$. This leads to the situation being identical to when the obedience constraint binds in the previous situation. The answer is thus identical where $t_{12} = \frac{(v_3-v_2)(\bar{v}-v^*)}{v_3x_2(v_2-v_1)}$ and $t_{13} = 1 - \frac{v_2(v^*-v_1)}{v_3x_3(v_2-v_1)}$.

In the next two scenarios, it is possible to achieve $u_0^*$ without putting any indication 3 patients in $s_1$, i.e. $t_{13} = 0$. What alters is which constraint binds. If $v^*$ is lower than $v_1x_1\left(\frac{v_2}{v_2-v_1}\right) + v_3x_3$ then the binding constraint is in $s_2$. Otherwise, it is in $s_1$.

$\bar{\bar{v}} < v^* < v_1x_1\left(\frac{v_2}{v_2-v_1}\right) + v_3x_3$ : Leads to $t_{12} = \frac{v_2x_2-u_0^*}{v_1x_2}$ and $t_{23} = \frac{v_2u_0^*-(v_2-v_1)v_2x_2}{v_1x_3(v_3-v_2)}$.

$v^* > v_1x_1\left(\frac{v_2}{v_2-v_1}\right) + v_3x_3$ : Leads to $t_{12} = \frac{v_1x_1}{(v_2-v_1)x_2}$ and $t_{23} = \frac{u_0^*-v_1x_1}{(v_3-v_2)x_3}$.

**Case 2:** $v_2(x_2 + x_3) < v_3x_3$

$v^* < \bar{v}$ : Again, we see that $t_{13} > 0$ is need to achieve $u_0^*$. So we get the variance minimising result $t_{12} = \frac{(v_3-v_2)(\bar{v}-v^*)}{v_3x_2(v_2-v_1)}$ and $t_{13} = 1 - \frac{v_2(v^*-v_1)}{v_3x_3(v_2-v_1)}$.

$\bar{v} < v^* < v_1x_1\left(\frac{v_2}{v_2-v_1)}\right) + v_3x_3$ : Means that $t_{12} = 0$, $t_{13} = \frac{u_0^*-v_2x_2}{(v_3-v_1)x_3}$ and $t_{23} = \frac{v_2x_2}{(v_3-v_2)x_3}$.

$\bar{\bar{v}} < v^* < v_1x_1\left(\frac{v_2}{v_2-v_1}\right) + v_3x_3$ : Leads to $t_{12} = \frac{v_2x_2-u_0^*}{v_1x_2}$ and $t_{23} = \frac{v_2u_0^*-(v_2-v_1)v_2x_2}{v_1x_3(v_3-v_2)}$.

$v^* > v_1x_1\left(\frac{v_2}{v_2-v_1}\right) + v_3x_3$ : Leads to $t_{12} = \frac{v_1x_1}{(v_2-v_1)x_2}$ and $t_{23} = \frac{u_0^*-v_1x_1}{(v_3-v_2)x_3}$.

**Situation 3:** $\frac{v_3}{2} > v_2$

In this case, out priority is to keep as many indication 2 patients in $s_1$ as possible without regard for minimising the number of indication patients in $s_1$. This is because the increase in variance from moving them to $s_2$ is higher than the variance from moving indication 3 patients to $s_1$.

So now our primary concern are the obedience constraints in $s_1$ and the probability constraint for indication 3.

$v^* < v_3(x_2 + x_3) - v_1x_1\left(\frac{v_3-v_2}{v_2-v_1}\right)$ : Leads to $t_{12} = \left(\frac{v_3-v_2}{v_2-v_1}\right)\frac{v_1x_1}{v_3x_2} + \frac{v^*-v_3x_3}{v_3x_2}$ and $t_{13} = 1 - \frac{v^*}{v_3x_3} + \frac{v_2v_1x_1}{v_3x_3(v_2-v_1)}$.

$v_3(x_2+x_3)-v_1x_1\left(\frac{v_3-v_2}{v_2-v_1}\right) < v^* < v_3x_3+v_2\frac{v_1x_1}{(v_2-v_1)}$ : Leads to $t_{12} = 1$ and $t_{13} = \frac{v_1(x_1+x_2)+v_3x_3-v^*}{(v_3-v_1)x_3}$.

$v^* > v_3x_3 + v_2\frac{v_1x_1}{(v_2-v_1)}$ : This is obviously the same as in the previous situation. The binding constraint is the one in $s_1$, $t_{12} = \frac{v_1x_1}{(v_2-v_1)x_2}$ and $t_{13} = 0$.

## 5.8 Insurance

We now look at the implications of segmentation during the monopoly period on markets with insurance. Once again, since prices are independent of segmentation in the competitive period there is nothing interesting to be said about its role there. [7]

[7]For those interested, Garcia and Tsur (2021) apply information design and segmentation to competitive insurance markets

If we assume the existence of an insurance model where the patient pays some fraction of the cost of treatment, while the insurer pays the rest. If the portion of the payment made by the patient is a function of a price $v_j$ then the payment made by the patient is $b(v_j)v_j$ while the insurer pays $(1 - b(v_j))v_j$. We explore both cases of whether the segmentation is done by the insurer or if it is done by an independent regulator.

If we now express the willingness to pay of the patient with indication $i$ as $a_i$ we get that any patient is willing to pay for a drug as long as $a_i \geq b(v_j)v_j$. If the insurer values indication $i$ at some amount $r_i$, then they are willing to pay for it if $r_i \geq (1 - b(v_j))v_j$. For treatment to be feasible it must be that $(a_i + r_i)x_i \geq c_i$. Thus, we can express $v_i = a_i + r_i$ as the total amount that the manufacturer can attain.

Furthermore, for now, we can assume that the contract entered into between the patient and the insurer is such that is captures the exact willingness to pay of both at each price $a_i = b(v_i)v_i$ and $r_i = (1 - b(v_i))v_i$. Thus $b(v_i)$ could be function that take different values for different indications.

Then the original non-negativity constraint gets replaced by the two non-negativity constraints for the patient and insurer respectively:

$$(b(v_i)v_i - b(v_j)v_j)t_{ji}x_i \geq 0 \quad \forall i, j$$

$$((1 - b(v_i))v_i - (1 - b(v_j))v_j)t_{ji}x_i \geq 0 \quad \forall i, j$$

The patient and insurer's surplus is then respectively,

$$u^*_{pat} = \max_{\{t_{ji}|i,j \in M\}} \sum_{i \in M} \sum_{j \in M} t_{ji}x_i((b(v_i)v_i - b(v_j)v_j))$$

$$u^*_{ins} = \max_{\{t_{ji}|i,j \in M\}} \sum_{i \in M} \sum_{j \in M} t_{ji}x_i((1 - b(v_i))v_i - (1 - b(v_j))v_j)$$

Since the total payment being made per indication is $v_i$ the obedience and marginal constraints remains the same as before. Assuming that the non-negativity constraints are met then $u^*_0 = u^*_{pat} + u^*_{ins}$.

### 5.8.1 Linear co-payments

If we assume that $b(v_j) = b$. Such that patients always pay a constant share of the total expenditure then both non-negativity constraints effectively collapse to the original single non-negativity constraint.

$b = 0$ leads to a particular case where all patients are fully insured and have zero co-payments. In this case, everything accrues to a single insurer and there are no issues of fairness to consider. We could even remove the non-negativity constraint where the insurer could occasionally pass off a low indication patient as a higher one. However, since everything accrues to the insurer and we already know the achievable sum this does not add much.

If $b(v) > 0$ then there is some portion of the surplus that accrues to the patients as $u^*_{pat} = bu^*_0$

and $u^*_{ins} = (1-b)u^*_0$. Consequently fairness considerations matter and I refer to the results from section 5.7. $b = 1$ is obviously the extreme case where there is effectively no insurance cover and we are in the free-market situation as described earlier.

**Lemma 12.** For linear co-payment functions the solutions for fairness are identical to the free-market case.

*Proof.* The proof follows quite obviously from the fact that the variance function for patients becomes

$$U_{pat} = b^2 \max_{\{t_{ji}|i,j \in M\}} \sum_{i \in M} \sum_{j \in M} t_{ji} x_i (v_i - v_j) - [bu^*_0]^2$$

which is simply a rescaling of the function in section 5.7. □

### 5.8.2 Non-linear co-payments

Now let's assume that co-payments are not a fixed fraction of the total cost of the drug. A state insurer, for example may protect patients from exorbitant costs by decreasing the share for more expensive drugs.

For now, we suppose there is a cap on expenditure such that $b(v)$ is a kinked function:

$$b(v) = \begin{cases} 1 & v \leq v_0 \\ \frac{v_0}{v} & v > v_0 \end{cases}$$

which implies that

$$b(v) = \begin{cases} 0 & v \leq v_0 \\ 1 - \frac{v_0}{v} & v > v_0 \end{cases}$$

Here we see that due to Theorem 3 we can achieve an efficient outcome, but the choice of objective function changes the outcome i.e. the insurer's optimisation would be different to the patients'.

Essentially, one can see that the insurer would look to maximise accrual of consumer surplus towards the indications covered by insurance while a regulator looking to maximise consumer surplus for patients would try to direct it towards indications not covered by insurance. Hence, we should see patients in higher value insured indications be moved towards lower value indications if the insurer does the segmentation. While patient surplus does not change for movement within indications above $v_0$ there should be no movement there and they can be considered a single indication for the purpose of consumer surplus. However, movement from higher indications to lower ones otherwise should be expected.

# 6 Conclusion and Discussion

## 6.1 Regulator Credibility and Robustness of Indication-Based Pricing

We have so far focussed on the problem of a regulator looking to maximise consumer surplus by controlling the manufacturer's information. We could flip the question in terms of establishing the robustness of Indication-based pricing, where we ask what the lower bound on profits would be to a manufacturer where patient indication would be completely hidden from them. If we assume a system with Indication-based pricing where the manufacturer deals directly with patients but has no way of verifying their indication then it is obvious from previous discussions that testing for all indications in $M^*$ would result in a profit $\pi^U_{M^*} < \pi^U_P$. Thus a manufacturer with foresight should achieve $\pi^U_P$ by only revealing $P$. Indication-based pricing is thus reliant on a regulator to reveal indications to the manufacturer.

If we now move towards Indication-based pricing with a regulator who imperfectly reveals the indication i.e. it sometimes lies about the indication to maximise consumer surplus. In a one-shot interaction with no commitment device for the regulator, we would see the same result where the regulator now lies on behalf of the patients in a way that reduces profit down to $\pi_{M^*}$ and the manufacturer responds as before. This is because it is not sequentially rational for the regulator to do otherwise without some sort of commitment device.

Let's now assume that the regulator and manufacturer are both playing a repeated game wherein the manufacturer develops a new drug in every stage game. Since we know that the manufacturer only reveals $I$ under Indication-based pricing, we know that $M = I$.

If they are both forward-looking then $\pi_I$ can serve as a Subgame Perfect Nash payoff for the manufacturer where it can always default to setting $M = P$ if the regulator lies too much. In this sense, we can interpret the regulator's behaviour in this paper as a robustness of the Indication-base pricing where a regulator's optimal strategy of lying satisfies the criterion set up in Theroem 1. This keeps the manufacturer from reverting to $M = P$ and maximises consumer surplus in the long run.

Returning to our original setup, we did not discuss the nature of the commitment for the algorithm from the regulator. One could assume it would be enforced by law. Here, I argue that it could hold simply as a strategy employed in a game of repeated interaction between the regulator and the manufacturer.

# 7 Extensions

## 7.1 Dynamics

Assume two time periods. In the first period, the manufacturer receives a certain set of indications to choose from $I$. In the second period a new indication, $N$, arrives which has a known valuation $v_N$ and which it can announce for cost, $c_N$. Previous indications are relabelled such that all indications, $i$ with valuation less than $v_N$ remain unchanged and those with greater are relabelled at $i + 1$. We refer to this updated set of indications as $I^+$.

We now consider two approaches towards updating the segments. On the one hand, we can treat this as we did the original static problem by updating $I$ and applying the same principle (i.e. the manufacturer is compensated for costs they would not have incurred in a uniform price set-up on top of the uniform monopoly profit they would have gained).

The other approach would be to recognise that once segmentation is done in the previous period, then the introduction of new indications can be incentivised simply by compensating for the introduction cost, $c_N$. This second approach may lead to strategic introduction in the first period as, based on the expected value of the new indication in the second period, $v_N$, the manufacturer may decide not to introduce any indications at all in the first period. Basically, since the payoff is based on the first set of indications announced by the manufacturer, they may be hesitant to launch the product if they anticipate the introduction of a new much more valuable indication in the future.

In either case, we establish the revenue that the manufacturer must be assured once the new indication appears.

The new revenue for the second period, $v^{*+}$, must be weakly greater than $v^*$.

We must find $P^+$. If $N$ is not in $P^+$ then there needs to be an additional compensation of $c_N$ i.e. $v^{*+} = v^* + c_N$ . For $c_N \in P^+$, this changes to $v^{*+} = v^* + (\pi^{*+} - \pi^*)$.

### 7.1.1 Finding $P^+$

We already know $K^r$ from our previous profit maximisation exercise. The addition of $N$ has no impact on $K^r$ if $r > N$. All $K^r$ for $r < N$ must be recalculated, and profits from each must be re-evaluated if there there has been a change from before.

For $K^N$, we know that if $i$ is in $K^{N-1}$ then it must exist in $K^N$. Furthermore, if $i$ doesn't exists in $K^{N+1}$ then it definitely doesn't exist in $K^N$. This narrows down the indications that need to be checked for $K^N$ to the contents of two sets. Thus, $K^N = K^{N-1} \cup \{i : i \in K^{N+1}, \tilde{c_i} < v_N\}$.

Profit for $K^1$ to $K^N$ needs to be re-calculated and compared with profit for $P^*$. If $\max\{\pi_{K^r} : r \leq N\}$ is less than $\pi_P$ then $v^{*+} = v^* + c_N$, otherwise $v^{*+} = v^* + (\max\{\pi_{K^r} : r \leq N\} - \pi_P)$

### 7.1.2 Updating Constraints

We need to update existing constraints and introduce new ones. We look to each type of constraints when an indication $N$ is added:

**Probability Constraints** Indication $i$ is associated with $i$ Probability Constraints. This include $i - 1$ non-negativity constraints and one marginal constraint. This leads to a total of $\frac{n(n+1)}{2}$ probability constraints (plus an identity $t_{ii} = 1 - \sum_{j<i} t_{ji}$ for each indication). The addition of indication $N$ consequently leads to the addition of $N$ new constraints. The constraints associated with $i < N$ remain unchanged, while marginal constraints for $i > N$ could potentially become tighter as there is one more $t_{Ni}$ involved; hence $n - N$ constraints have to change. This is the equivalent of saying that there is now another segment that could take in patients with indication $i$ leading to $s_i$ not existing. Such a possibility does not exist for indications whose valuations are lower than $N$ as those patients cannot be efficiently placed in $s_N$.

**Obedience Constraints** Each indication $i$ is associated with $n - i$ obedience constraints. This results in $\frac{n(n-1)}{2}$ total obedience constraints. Hence we see an addition of $N - i$ new constraints. The constraints for $i < N$ that prevent deviation to another price $j < i < N$ become tighter because there are more patients available and consequently greater incentive to charge a higher price. Deviations to $j > N$ become more lax as there there are new patients available who are willing to buy at $i$ but not at $j$. Furthermore, the constraints associated with

$i > N$ are unchanged as the new patients won't feature in this consideration since they will not buy at any price above $v_N$.

**General Updating Algorithm** In any case, the addition of any new indication can be made feasible through keeping the original segmentation, and adding a new segment for the new indication. This would imply a complete capture of the increase in surplus from addition of this new indication. This is an extreme case of compensation for the manufacturer. In most cases, patients can then be moved from or to this segment from others in order to reach, based on which constraints bind. This seems the least intrusive way of carrying out an addition, but says nothing about implications for distributional considerations on consumer surplus. The least intrusive way of achieving those targets need more work.

## 7.2 Minimising Administrative cost

Supposing a higher number of groups leads to more complexity and consequently higher costs, the regulator may also be interested in minimizing the number of groups. This can be done by reformulating the marginal constraints as

$$\sum_{j \in M^*} t_{ji} \leq 1 + 2(1 - z_j) \forall i$$

$$\sum_{j \in M^*} t_{ji} \geq z_j \forall i$$

and setting the objective function to $U() = \max \sum_j z_j$.

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# A Three-Indication Example

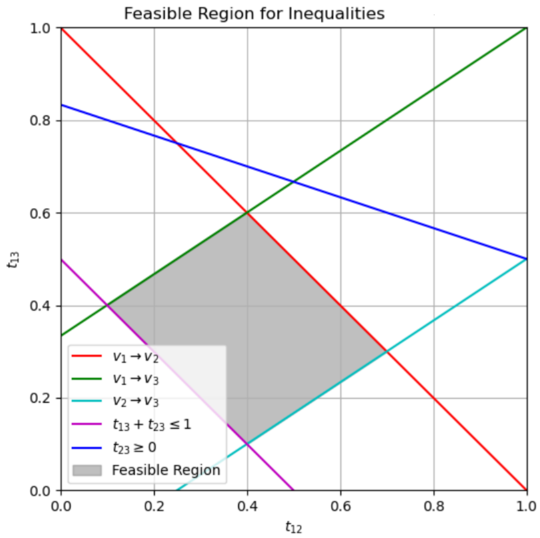



Figure 5: Feasible set and constraints for $m = 3$

In this section I present a graphical explanation for the results in section 5.7 for the case of three indications.

The obedience constraints can then be expressed as

$$t_{13} \leq \frac{v_1 x_1}{(v_2 - v_1)x_3} - \frac{x_2}{x_3} t_{12}$$

$$t_{13} \leq \frac{v_1 x_1}{(v_3 - v_1)x_3} + \frac{v_1 x_2}{(v_3 - v_1)x_3} t_{12}$$

$$t_{13} \geq \frac{u_0^* - v_2 x_2}{(v_3 - v_1)x_3} + \frac{v_1 x_2}{(v_3 - v_1)x_3} t_{12} = \frac{v_1 x_1 + v_3 x_3 - v^*}{(v_3 - v_1)x_3} + \frac{v_1 x_2}{(v_3 - v_1)x_3} t_{12}$$

the marginal constraints ($t_{13} + t_{23} \leq 1$):

$$t_{13} \geq \frac{u_0^* - (v_3 - v_2)x_3}{(v_2 - v_1)x_3} - \frac{x_2}{x_3} t_{12} = \frac{v_1 x_1 + v_2(x_2 + x_3) - v^*}{(v_2 - v_1)x_3} - \frac{x_2}{x_3} t_{12}$$

and the probability constraints: ($t_{13}, t_{23}, t_{23} \geq 0$)

$$1 \geq t_{12} \geq 0$$

$$t_{13} \geq 0$$

$$t_{13} \leq \frac{u_0^*}{(v_3 - v_1)x_3} - \frac{(v_2 - v_1)x_2}{(v_3 - v_1)x_3} t_{12} = \frac{v_1 x_1 + v_2 x_2 + v_3 x_3 - v^*}{(v_3 - v_1)x_3} - \frac{(v_2 - v_1)x_2}{(v_3 - v_1)x_3} t_{12}$$

.

Figure 5 shows the constraints that define the feasible set of solutions. These constraints move based on the $V$, $x$ and the target expected revenue $v^*$, but retain the same basic structure; two parallel downward sloping lines and two parallel upward sloping ones, and one downward sloping constraint.

The gradient vector for $U_k$ is given by $\overline{U}_k$.

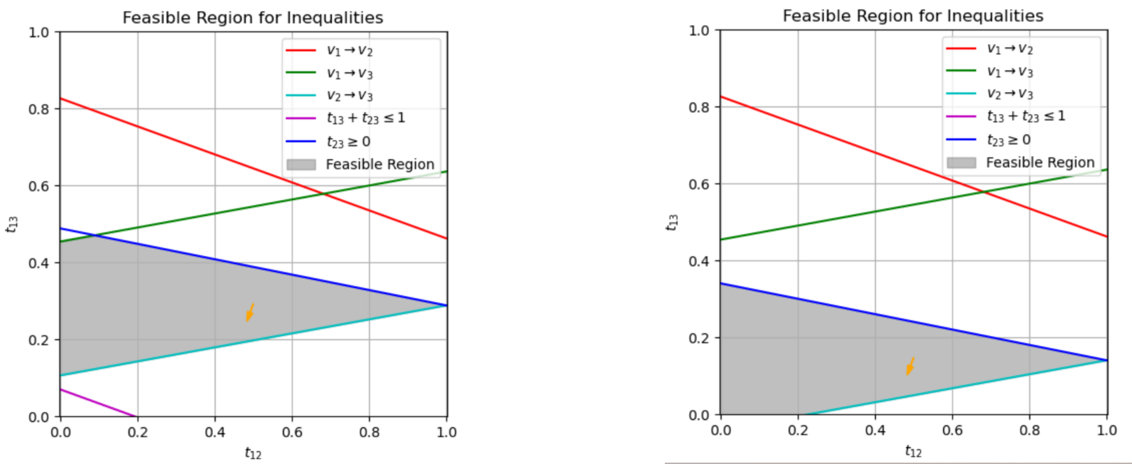
$$\overline{U}_1 = (v_2 - v_1)\begin{bmatrix}(2v_2 - v_3 - v_1)x_2 \\ (v_3 - v_1)x_3\end{bmatrix}$$

$$\overline{U}_2 = (v_2 - v_1)(v_3 - v_1)\begin{bmatrix}x_2 \\ x_3\end{bmatrix}$$

### Price Variance minimisation

In order to minimize each $U$, we need to move in the direction of $-\overleftarrow{U}$. We can see now that the normal to the marginal constraint is $\begin{bmatrix}x_2 \\ x_3\end{bmatrix}$. This is in the exact opposite direction to $-\overline{U}_2$, which mean that variance is minimised on the marginal constraint i.e. where $t_{13} + t_{23} = 1$. There are two cases when this is not achievable; when the constraint lies below the first quadrant or when the obedience constraint $v_2 \to v_3$ supercedes it.

Figure 6 shows the two situations in which $t_{13} + t_{23} = 1$ is not possible. The yellow arrow shows the direction in which price variance is minimised. Explanations for the intuition for the occurrence of each situation is given in section 5.7.




(a) Feasible set and constraints for $m = 3$

(b) Feasible set and constraints for $m = 3$

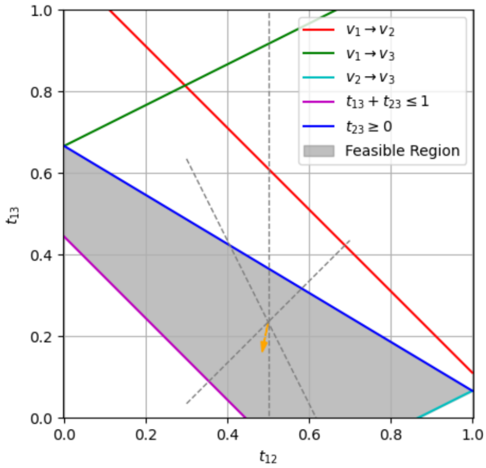
Figure 6: Feasible sets and constraints for $m = 3$: (a) and (b)

### Consumer Surplus Variation minimisation

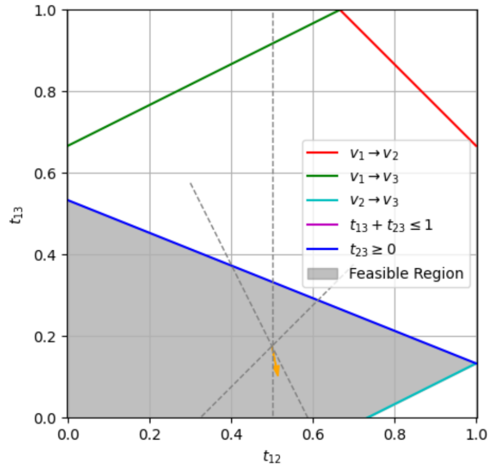
$t_{13}$ needs to always be reduced to minimise $U_1$. However, the sign on $2v_2 - v_3 - v_1$ determines whether increasing $t_{12}$ minimizes $U_1$ or not i.e. if $2v_2 - v_3 - v_1 > 0$ then minimising consumer surplus variation requires reducing $t_{12}$, otherwise, it needs to be increased.

When $2v_2 - v_3 - v_1 > 0$, we know that $2v_2 - v_3 - v_1 < v_3 - v_1$, so minimising consumer surplus coincides with price variance minimisation; if $t_{13} + t_{23} = 1$ is feasible, then consumer surplus is minimised on the constraint with the lowest possible value of $t_{13}$. So both solutions coincide in all cases, but for consumer surplus it is unique in all cases.

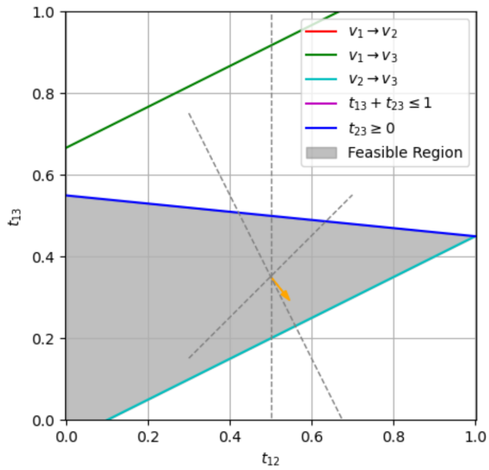
For $2v_2 - v_3 - v_1 < 0$, the relevant constraints lie on the bottom right. So we check the normal to the $v_2 \to v_3$ constraint, $\begin{bmatrix} -v_1 x_2 \\ (v_3 - v_1)x_3 \end{bmatrix}$. If $-(2v_2 - v_3 - v_1) < v_1$ then $-\overline{U}_1$ lies below the normal. This corresponds to $v_2 > \frac{v_3}{2}$ so the solution corresponds to the lowest possible $t_{13}$ while for $v_2 < \frac{v_3}{2}$ we get a higher $t_{13}$. These results are illustrated in figures. The reasoning is established in the main text in section 5.7.




(a) $\frac{v_1+v_3}{2} < v_2$




(b) $\frac{v_3}{2} < v_2 < \frac{v_1+v_3}{2}$




(c) $v_2 < \frac{v_3}{2}$

Figure 7: Binding Constraints based on $V$

Figure 7 shows the three cases that can occur and the implications for the constraints that bind. The arrow shows the direction in which Consumer Surplus Variance is minimised, while the dashed lines show normals to the constraints $t_{13} + t_{23} \leq 1$, $v_2 \to v_3$ and $t_{13} \geq 0$. We thus identify the three corners at which the solution could exist, and consequently the binding constraints, based on values of $V$. For the case where $v_2 > \frac{v_1+v_3}{2}$, this is the bottom left, where variance is minimised and the constraint that must bind is either $t_{13} + t_{23} \leq 1$ or $t_{12} \geq 0$. The second constraint needed to define the corner is either $v_2 \to v_3$ or $t_{13} \geq 0$. For $v_2 < \frac{v_1+v_3}{2}$, the corner is the bottom right, with the constraint $v_2 \to v_3$ always binding. The second constraint that binds (to form the corner) is driven by whether $v_2 < \frac{v_3}{2}$ or $v_2 < \frac{v_3}{2}$.

The cases built in section 5.7 are built around the possible shapes the feasible set can take and the consequent binding constraints.